$2^p - 1$

Mersenne

M_{44}

$2^{2^n} - 1$

Fermat

Public Key Encryption

$(2^n - 1)^2 - 2$

A Prime Experience

$\frac{m^3 - n^3}{m - n}, m = n + 1$

$k \times 2^n + 1$

Sexy Primes

$\frac{2^n + 1}{3}$

Prince Alfred College
Founded 1869

Noel Baker Centre
for School Mathematics

CASIO®

A Prime Experience.

Version 1.00 – May 2008.

Written by Anthony Harradine and Alastair Lupton

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Index

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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Legend.

EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just explore and think!

At certain points the learning process should have generated some **burning mathematical questions** that should be discussed and pondered, and then answered as you learn more!

Time to Formalise.

These notes document the learning that has occurred to this point, using a degree of formal mathematical language and notation.

Examples.

Illustrations of the mathematics at hand, used to answer questions.

1. What do you know about Primes?

1. What are the properties of prime numbers?

2. What name is given to integers that are not prime?

3. List as many primes as you can, in order

The Fundamental Theorem of Arithmetic states that every integer greater than 1 can be written as the product of one or more primes, and that this representation is unique (except for the order of the primes)[1]

A number's prime factorisation is its (only) unique representation, and so can be thought of as its mathematical finger print.

To obtain a prime factorisation, factor, and then factor the factors, until you can factor no more (without using 1). It doesn't matter which factors you choose, you will always end up with the same set of prime factors!

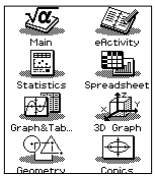
$$
\begin{aligned}
&1400 \\
&= 14 \times 100 \\
&= 2 \times 7 \times 10 \times 10 \\
&= 2 \times 7 \times 2 \times 5 \times 2 \times 5 \\
&= 2 \times 2 \times 2 \times 5 \times 5 \times 7 \\
&(= 2^3 \times 5^2 \times 7)
\end{aligned}
\qquad
\begin{aligned}
&1400 \\
&= 2 \times 700 \\
&= 2 \times 2 \times 350 \\
&= 2 \times 2 \times 50 \times 7 \\
&= 2 \times 2 \times 2 \times 25 \times 7 \\
&= 2 \times 2 \times 2 \times 5 \times 5 \times 7
\end{aligned}
$$

4. Obtain the unique prime factorisations of the following integers

a.	70	b.	36	c.	400
d.	16	e.	210	f.	363
g.	648	h.	5439	i.	113

5. If you needed to know whether or not 19043 was prime, what method could you employ to find out?

6. If you have not already done so, continue your list of primes so that it includes at least the 25 that are below 100.



5.1

[1] One of the implications of this theorem is that *1 cannot be a prime number*. If it was then $30 = 2 \times 3 \times 5$ and $30 = 1 \times 2 \times 3 \times 5$ would provide a counter argument that would disprove this theorem in its current format.

2. *Arithmetic Sequences of Prime Numbers.*

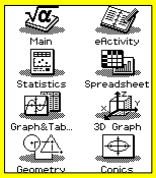
Whilst appearing somewhat random in their distribution, much study has gone the patterns that are evident within the set of prime numbers.

One such pattern is the occurrence of *arithmetic sequences of prime numbers.*[2]

EAT 1

2.1 How many arithmetic sequences of primes can you find?

What is the longest one you can find?



5.2

Do you notice anything about the sequences that you find?

2.1. Sexy Primes.

Groups of prime numbers that differ by 6 are known as *sexy primes.* [3]

EAT 2

Can you find 10 pairs of sexy primes?

Can you find a sexy prime triple?

What about a sexy prime quadruple?

What is the largest group of sexy primes that you can find?

Do you think there might be larger groups of sexy primes?

---

[2] Arithmetic sequences are ordered sets of numbers where consecutive members differ by a fixed value e.g. 7,15,23,31 and 43,61,79. The second of these is an arithmetic sequence of prime numbers with a length of 3.

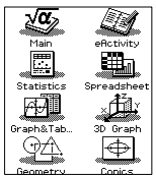
[3] The name is derived from *sex* – the latin word for six.

3. *Generating Prime Numbers.*

The existence of prime numbers was known by Egyptian mathematicians. Prime numbers were extensively studied by the ancient Greeks over 2000 years ago. Despite this, the aspects of number theory that deal with prime numbers are amongst the most active in modern mathematics. One reason for this is that no one has ever found an explicit way of generating primes without missing any or producing non-primes.

Attempts at prime generation.

Named after Marin Mersenne, a 17th century French scholar, Mersenne Primes are generated by $2^p - 1$ where p is prime



5.3

1. Generate the 8 Mersenne Primes that are less than 10^{18}.

As of January 2008, 44 Mersenne Primes are known.

The 44th Mersenne Prime (known as M_{44}) has 9 808 358 digits.

The finders of the 45th Mersenne Prime will very likely win the Electronic Frontier Federation's US$100,000 prize for the first finder of a prime with over 10 million digits.[4]

Named after Pierre de Fermat, Fermat Primes are generated by $2^{2^n} + 1$, with integer n. In 1650 Fermat made the conjecture that all such numbers were prime.

2. Find the 5 Fermat Primes
3. Prove that Fermat was wrong.

Other prime generators

On the follow page is a list of some (but not all) of the known prime number generators.

For as many of the prime number generators as possible

4. Use the generator to generate as many primes as you can (max. of 10)
5. Show that the generator does not *exclusively* generate primes.
6. Show that the generator does not generate all the primes.

---

[4] The E.F.F. are also offering US$150,000 for the a prime with more than 100 million digits and US$250,000 for a prime with more than a billion digits.

Prime Number Generators

In all cases n and m are *natural numbers* (integers including zero)

Carol Primes	$(2^n - 1)^2 - 2$
Centred square primes	$n^2 + (n+1)^2$
Cuban Primes	$\frac{m^3 - n^3}{m - n}$, $m = n + 1$
Gaussian Primes	$4n + 3$
Kynea Primes	$(2^n + 1)^2 - 2$
Leyland Primes	$n^m + m^n$ with $1 < n \leq m$
Pierpoint Primes	$2^n 3^m + 1$
Proth Primes	$k \times 2^n + 1$ with odd k and $k < 2^n$ [5]
Pythagorean Primes	$4n + 1$
Star Primes	$6n(n-1) + 1$
Thabit number Primes	$3 \times 2^n - 1$
Wagstaff Primes	$\frac{2^n + 1}{3}$

Why generate primes?

As well of its academic desirability, the ability to generate very large prime numbers is a very valuable skill. This is because very large primes are essential to *Public Key Encryption*, the style of encryption that protects much of the world's internet-based communication and commerce.

At the heart of Public Key Encryption like the RSA cipher is the multiplication of two very large prime numbers. The security of this cipher relies on the fact that it is very difficult to factorise the product of two very large prime numbers.

At the moment RSA ciphers use numbers with in excess of 200 digits that are generated by the multiplication of primes with roughly 100 digits. 10 years ago the primes used had 'only' 50 to 60 digits. As available computer power grows, the encryption that keeps electronic commerce secure has to keep up, with larger and larger primes. This is what gives primes their commercial value.

---

[5] The largest known Non-Mersenne Prime is the Proth Prime $19249 \times 2^{13,018,586} + 1$ which has 3 918 990 digits.

4. *Going Further.*

4.1 So many sexy primes …

The existence of so many sexy primes suggests that arithmetic sequences with common differences of 6 have a particular relationship with prime numbers.

EAT 4

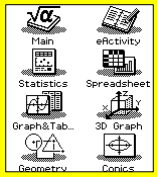
Investigate arithmetic sequences of the form $6n + m$ (where $n = 0, 1, 2, \ldots$)

with a range of integer values for the constant m

Which of these sequences generates the most primes?

How many primes are generated by these sequences?

Why do the others not generate as many primes?



5.4

4.2 Proofs required.

1. In **EAT 2**, you should have come up with the *only* sexy prime quintuple. Prove that there can be no other.
2. In **EAT 4** you should have found the two arithmetic sequences that seem to generate *all the prime numbers greater than 3*. Prove that they do.
3. While you are at it, prove that there is no integer p such that the set $\{p,\ p+8,\ p+22\}$ is prime.

4.3 The big generalisation – a big challenge.

As of 7/5/08, the *Wikipedia* entry on prime numbers contained the following claim

> all prime numbers above *q* are of form $q\# \times n + m$,
> where $0 < m < q$, and *m* has no prime factor ≤ *q*. [6]

There is an error in this statement!

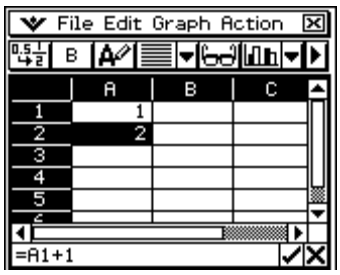
By studying and trying to apply this fact to the primes that you know, locate the error/omission.

---

[6] For $n \geq 2$, the **primorial** ($q\#$) is the product of all prime numbers less than or equal to *q*. For example, 7# = 210 is a primorial which is the product of the first four primes multiplied together ($2 \times 3 \times 5 \times 7$).

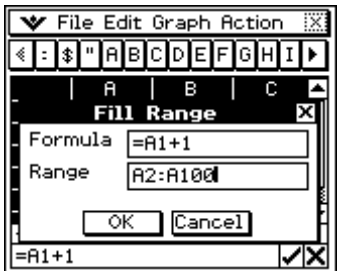
5. eTech Support.

5.1 Finding primes amongst integers.

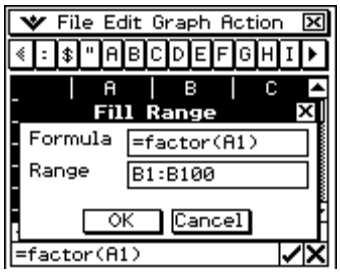


To use a ClassPad 300 spreadsheet to find primes, we first need a list of integers.

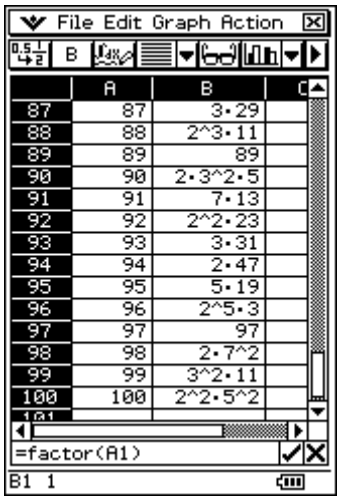
These can be obtained by entering the number `1` in cell `A1` and then the formula `= A1+1` in cell `A2`.
With cell `A2` selected (bold), tap on the `Edit` menu and then tap on `Fill Range`.



The formula from `A2` will not need changing, but the range can be extended to `A2:A100` (or beyond) to obtain as many integers as required.



To see which of these integers are prime, we need to determine their prime factorisation.
Entering in `B1` the command `= factor(A1)` will do this to `A1`, and the `Fill Range` command can be used to fill this command down to `B100`, prime factoring the first 100 integers.

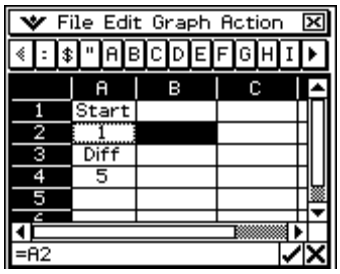


If you want to obtain more primes, just change the value in `A1` to 101 (for example), and you will have the prime factorisation of the next 100 integers.
This will help you identify the 21 primes that lie between between 101 and 200.

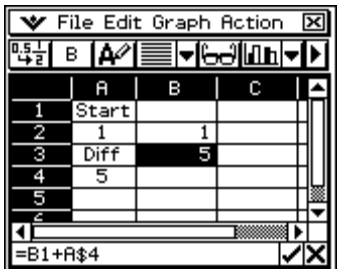
Notation Alert

The ClassPad uses a dot to indicate multiplication. This dot is placed a little higher then a decimal point. It is obviously important not to confuse a dot indicating multiplication with a decimal point.

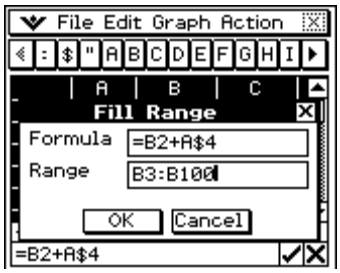
5.2 Finding primes amongst arithmetic sequences



One way to find arithmetic sequences of primes is to first generate arithmetic sequences, and then find the primes amongst them. To generate an arithmetic sequence we will modify the method used previously, to make a more easily modified spreadsheet. To start with, define the initial value (`Start`) and common difference (`Diff`) that will specify our arithmetic sequence (in column `A` as shown).

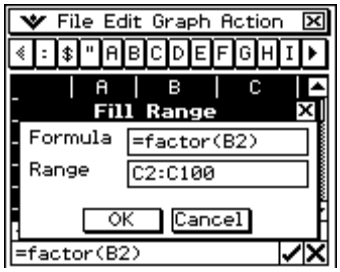


In cell `B2` enter the formula `=A2` (to start our sequence) and in the cell `B3` enter the formula `=B1+A$4`. The dollar (`$`) sign will ensure that this formula always refers to the common difference that is stored in cell `A4`.

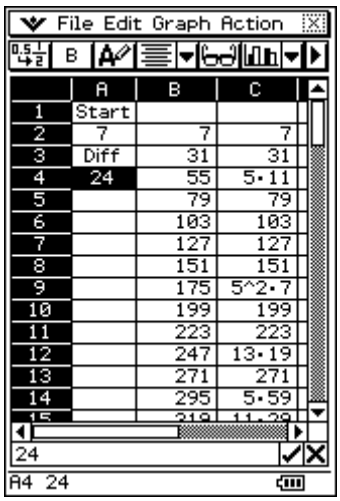


Use the `Fill Range` command (from the `Edit` menu) to fill this formula down to `B100`.

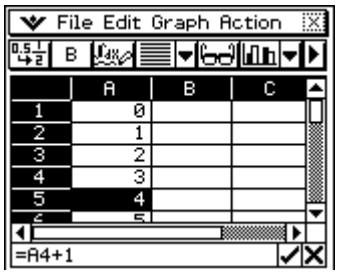
To determine the members of your arithmetic sequence that are prime, use the `factor` command.
As previously, this can be entered into cell `C2` as `= factor(B2)`, and then the `Fill Range` command can be used to fill the cells `C2:C100` with this command.



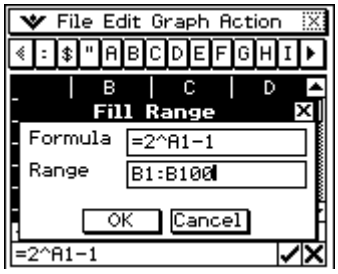
With the spreadsheet built (as shown left) the `Start` value (in `A2`) and Common Difference (`Diff` in `A4`) can be modified to search amongst a range of arithmetic sequences to find arithmetic sequences of primes. Shown left is the sequence generated by $24n+7$ where $n=0,1,2,...$ As you can see, this has uncovered a number of arithmetic sequences including $\{79,103,127,151\}$ and $\{7,79,151,223\}$ as well as several shorter sequences.



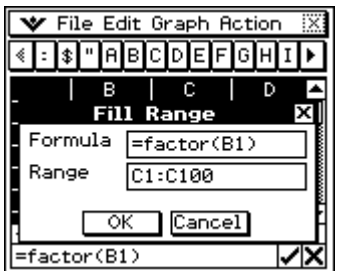
5.3 Using Prime Number Generators.



Using a spreadsheet with a column of integers $n = 0,1,2,...$ (see **eTech Support 5.1** for help with this) we can enter the prime number generating formula into `B1` as `=2^A1-1` (or whatever formula we are using to generate primes) and fill down using the `Fill Range` command in the `Edit` menu.



To see whether or not the numbers generated are prime or not, we will once again use the `factor` command i.e. `=factor(B1)` in cell `C1` and `Fill Range` for cells `C1:C100`.



The screen shot below is taken from *ClassPad Manager (version 3) Professional*, a PC-based emulation software package. In it we can see $M_8 = 2147483647$.

File Edit Graph Action

	A	B	C
14	13	8191	8191
15	14	16383	3·43·127
16	15	32767	7·31·151
17	16	65535	3·5·17·257
18	17	131071	131071
19	18	262143	3^3·7·19·73
20	19	524287	524287
21	20	1048575	3·5^2·11·31·...
22	21	2097151	7^2·127·337
23	22	4194303	3·23·89·683
24	23	8388607	47·178481
25	24	16777215	3^2·5·7·13·1...
26	25	33554431	31·601·1801
27	26	67108863	3·2731·8191
28	27	134217727	7·73·262657
29	28	268435455	3·5·29·43·11...
30	29	536870911	233·1103·2089
31	30	1073741823	3^2·7·11·31·...
32	31	2147483647	2147483647
33	32	4294967295	3·5·17·257·6...

=factor(B32)

C32 2147483647

Note: Mersenne Primes have to

(a) be prime and

(b) have prime *p* in $2^p - 1$.

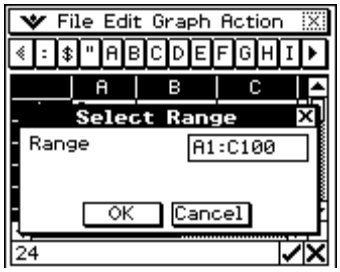
This is the case for $p = 2,3,5,7,13,17,19,31$, the values that generate the first eight Mersenne Primes.

5.4 Searching multiple arithmetic sequences.

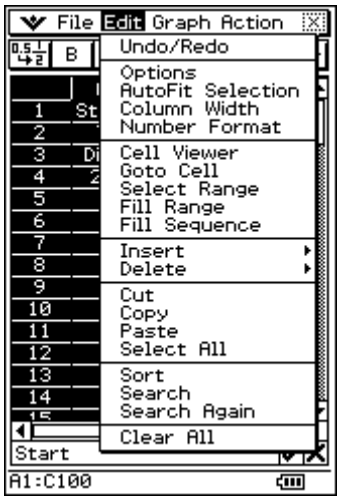
The arithmetic sequence spreadsheet will clearly be useful for searching for sexy primes, as these differ from each other by 6.

One trick that you may want to use is to duplicate the active cells of the spreadsheet so that you can view a number of arithmetic sequences at once.

To do this select the desired cells i.e. `A1:C100`.
One efficient way to do this is via the `Select Range` command that is available in the `Edit` menu.

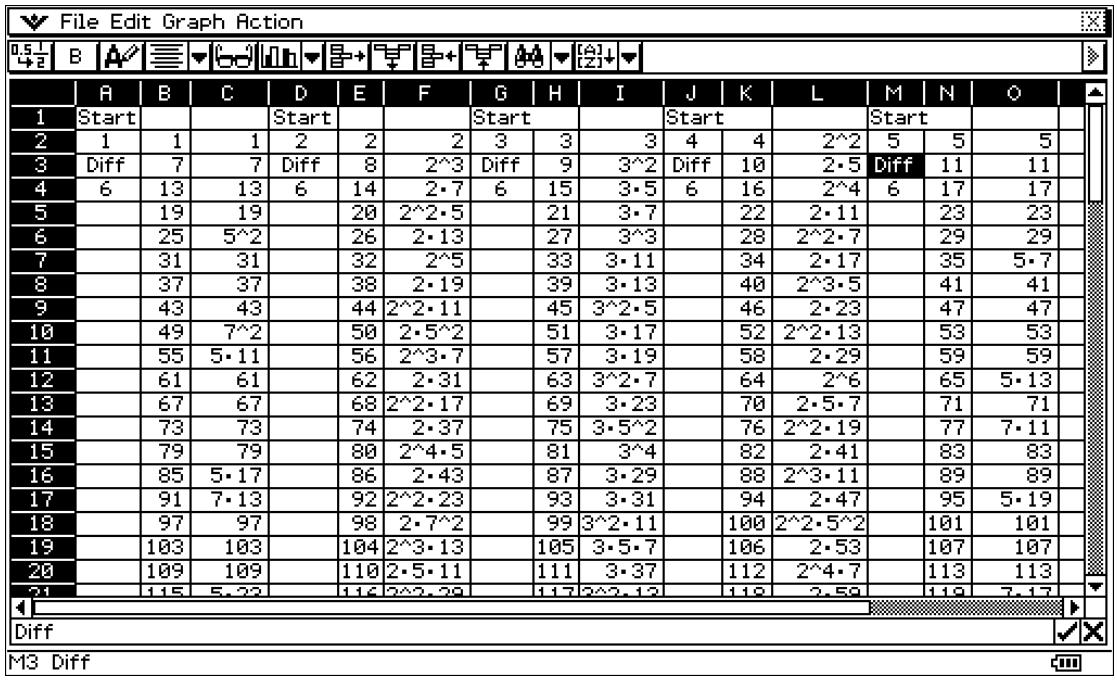


With the desired cells selected, tap `Copy` (also available from the `Edit` menu), then tap in the top left corner of the chosen destination cells (in this case `D1`) and then tap `Paste` (also from the `Edit` menu).



As your spreadsheet gets larger you can see more of it by narrowing the columns. This can be done by dragging the column divisions in the *column label row* to the left or right (see below).

The screen shot below is taken from *ClassPad Manager (version 3) Professional*, a PC-based emulation software package. It shows 5 arithmetic sequences being investigated.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Start			Start			Start			Start			Start		
2	1	1	1	2	2	2	3	3	3	4	4	2^2	5	5	5
3	Diff	7	7	Diff	8	2^3	Diff	9	3^2	Diff	10	2·5	Diff	11	11
4	6	13	13	6	14	2·7	6	15	3·5	6	16	2^4	6	17	17
5		19	19		20	2^2·5		21	3·7		22	2·11		23	23
6		25	5^2		26	2·13		27	3^3		28	2^2·7		29	29
7		31	31		32	2^5		33	3·11		34	2·17		35	5·7
8		37	37		38	2·19		39	3·13		40	2^3·5		41	41
9		43	43		44	2^2·11		45	3^2·5		46	2·23		47	47
10		49	7^2		50	2·5^2		51	3·17		52	2^2·13		53	53
11		55	5·11		56	2^3·7		57	3·19		58	2·29		59	59
12		61	61		62	2·31		63	3^2·7		64	2^6		65	5·13
13		67	67		68	2^2·17		69	3·23		70	2·5·7		71	71
14		73	73		74	2·37		75	3·5^2		76	2^2·19		77	7·11
15		79	79		80	2^4·5		81	3^4		82	2·41		83	83
16		85	5·17		86	2·43		87	3·29		88	2^3·11		89	89
17		91	7·13		92	2^2·23		93	3·31		94	2·47		95	5·19
18		97	97		98	2·7^2		99	3^2·11		100	2^2·5^2		101	101
19		103	103		104	2^3·13		105	3·5·7		106	2·53		107	107
20		109	109		110	2·5·11		111	3·37		112	2^4·7		113	113
21		115	5·23		116	2^2·29		117	3^2·13		118	2·59		119	7·17

6. *Appendix.*

The first 1000 prime numbers are:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109
113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199 211 223 227 229
233 239 241 251 257 263 269 271 277 281 283 293 307 311 313 317 331 337 347 349 353
359 367 373 379 383 389 397 401 409 419 421 431 433 439 443 449 457 461 463 467 479
487 491 499 503 509 521 523 541 547 557 563 569 571 577 587 593 599 601 607 613 617

619 631 641 643 647 653 659 661 673 677 683 691 701 709 719 727 733 739 743 751 757
761 769 773 787 797 809 811 821 823 827 829 839 853 857 859 863 877 881 883 887 907
911 919 929 937 941 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033
1039 1049 1051 1061 1063 1069 1087 1091 1093 1097 1103 1109 1117 1123 1129 1151
1153 1163 1171 1181 1187 1193 1201 1213 1217 1223 1229 1231 1237 1249 1259 1277

1279 1283 1289 1291 1297 1301 1303 1307 1319 1321 1327 1361 1367 1373 1381 1399
1409 1423 1427 1429 1433 1439 1447 1451 1453 1459 1471 1481 1483 1487 1489 1493
1499 1511 1523 1531 1543 1549 1553 1559 1567 1571 1579 1583 1597 1601 1607 1609
1613 1619 1621 1627 1637 1657 1663 1667 1669 1693 1697 1699 1709 1721 1723 1733
1741 1747 1753 1759 1777 1783 1787 1789 1801 1811 1823 1831 1847 1861 1867 1871

1873 1877 1879 1889 1901 1907 1913 1931 1933 1949 1951 1973 1979 1987 1993 1997
1999 2003 2011 2017 2027 2029 2039 2053 2063 2069 2081 2083 2087 2089 2099 2111
2113 2129 2131 2137 2141 2143 2153 2161 2179 2203 2207 2213 2221 2237 2239 2243
2251 2267 2269 2273 2281 2287 2293 2297 2309 2311 2333 2339 2341 2347 2351 2357
2371 2377 2381 2383 2389 2393 2399 2411 2417 2423 2437 2441 2447 2459 2467 2473

2477 2503 2521 2531 2539 2543 2549 2551 2557 2579 2591 2593 2609 2617 2621 2633
2647 2657 2659 2663 2671 2677 2683 2687 2689 2693 2699 2707 2711 2713 2719 2729
2731 2741 2749 2753 2767 2777 2789 2791 2797 2801 2803 2819 2833 2837 2843 2851
2857 2861 2879 2887 2897 2903 2909 2917 2927 2939 2953 2957 2963 2969 2971 2999
3001 3011 3019 3023 3037 3041 3049 3061 3067 3079 3083 3089 3109 3119 3121 3137

3163 3167 3169 3181 3187 3191 3203 3209 3217 3221 3229 3251 3253 3257 3259 3271
3299 3301 3307 3313 3319 3323 3329 3331 3343 3347 3359 3361 3371 3373 3389 3391
3407 3413 3433 3449 3457 3461 3463 3467 3469 3491 3499 3511 3517 3527 3529 3533
3539 3541 3547 3557 3559 3571 3581 3583 3593 3607 3613 3617 3623 3631 3637 3643
3659 3671 3673 3677 3691 3697 3701 3709 3719 3727 3733 3739 3761 3767 3769 3779

3793 3797 3803 3821 3823 3833 3847 3851 3853 3863 3877 3881 3889 3907 3911 3917
3919 3923 3929 3931 3943 3947 3967 3989 4001 4003 4007 4013 4019 4021 4027 4049
4051 4057 4073 4079 4091 4093 4099 4111 4127 4129 4133 4139 4153 4157 4159 4177
4201 4211 4217 4219 4229 4231 4241 4243 4253 4259 4261 4271 4273 4283 4289 4297
4327 4337 4339 4349 4357 4363 4373 4391 4397 4409 4421 4423 4441 4447 4451 4457

4463 4481 4483 4493 4507 4513 4517 4519 4523 4547 4549 4561 4567 4583 4591 4597
4603 4621 4637 4639 4643 4649 4651 4657 4663 4673 4679 4691 4703 4721 4723 4729
4733 4751 4759 4783 4787 4789 4793 4799 4801 4813 4817 4831 4861 4871 4877 4889
4903 4909 4919 4931 4933 4937 4943 4951 4957 4967 4969 4973 4987 4993 4999 5003
5009 5011 5021 5023 5039 5051 5059 5077 5081 5087 5099 5101 5107 5113 5119 5147

5153 5167 5171 5179 5189 5197 5209 5227 5231 5233 5237 5261 5273 5279 5281 5297
5303 5309 5323 5333 5347 5351 5381 5387 5393 5399 5407 5413 5417 5419 5431 5437
5441 5443 5449 5471 5477 5479 5483 5501 5503 5507 5519 5521 5527 5531 5557 5563
5569 5573 5581 5591 5623 5639 5641 5647 5651 5653 5657 5659 5669 5683 5689 5693
5701 5711 5717 5737 5741 5743 5749 5779 5783 5791 5801 5807 5813 5821 5827 5839

5843 5849 5851 5857 5861 5867 5869 5879 5881 5897 5903 5923 5927 5939 5953 5981
5987 6007 6011 6029 6037 6043 6047 6053 6067 6073 6079 6089 6091 6101 6113 6121
6131 6133 6143 6151 6163 6173 6197 6199 6203 6211 6217 6221 6229 6247 6257 6263
6269 6271 6277 6287 6299 6301 6311 6317 6323 6329 6337 6343 6353 6359 6361 6367
6373 6379 6389 6397 6421 6427 6449 6451 6469 6473 6481 6491 6521 6529 6547 6551

6553 6563 6569 6571 6577 6581 6599 6607 6619 6637 6653 6659 6661 6673 6679 6689
6691 6701 6703 6709 6719 6733 6737 6761 6763 6779 6781 6791 6793 6803 6823 6827
6829 6833 6841 6857 6863 6869 6871 6883 6899 6907 6911 6917 6947 6949 6959 6961
6967 6971 6977 6983 6991 6997 7001 7013 7019 7027 7039 7043 7057 7069 7079 7103
7109 7121 7127 7129 7151 7159 7177 7187 7193 7207 7211 7213 7219 7229 7237 7243

7247 7253 7283 7297 7307 7309 7321 7331 7333 7349 7351 7369 7393 7411 7417 7433
7451 7457 7459 7477 7481 7487 7489 7499 7507 7517 7523 7529 7537 7541 7547 7549
7559 7561 7573 7577 7583 7589 7591 7603 7607 7621 7639 7643 7649 7669 7673 7681
7687 7691 7699 7703 7717 7723 7727 7741 7753 7757 7759 7789 7793 7817 7823 7829
7841 7853 7867 7873 7877 7879 7883 7901 7907 7919

7. Answers.

1. What do you ….

Qu. 1
Prime numbers are

- Positive integers
- With *exactly* 2 (positive integer) divisors/factors, 1 and themselves

Qu. 2

Composite

Qu. 3

2, 3, 5, 7, 11, 13, 17, 19, 23, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Qu. 4

a. $70 = 2 \times 5 \times 7$

b. $36 = 2 \times 2 \times 3 \times 3$

c. $400 = 2^4 \times 5^2$

d. $16 = 2^4$

e. $210 = 2 \times 3 \times 5 \times 7$

f. $363 = 3 \times 11 \times 11$

g. $648 = 2^3 \times 3^4$

h. $5439 = 3 \times 7 \times 7 \times 37$

i. $113 = 113$

Qu. 5

You could try dividing 19043 by the primes starting with 2, 3, 5, … You would need keep going until you reached a divisor bigger than $\sqrt{19043}$ (i.e. 138).

You would know you had reached this point when the result of the division was less than the divisor used.

In this case however you would not get to this point as

$$\frac{19043}{137} = 139$$

So 19043 is not prime, as it has 137 and 139 as factors.

Unfortunately, there are 32 primes to check (as divisors) until you reach 137!

Qu. 6

See Question 3

2. Generating Primes….

Qu. 1
3, 7, 31, 127, 8191, 131071, 524287, 2147483647.

Qu. 2

3, 5, 17, 257, 65537

Qu. 3

$$2^{2^5} + 1 = 4294967297$$

$$4294967297 = 641 \times 6700417$$

Qu. 4, 5, 6

Carol Primes

$(2^n - 1)^2 - 2$

Generator gives
7, 47, 223, 3967,16127, 1046527, 16769023, 1073676287, …
which are prime

Generator also gives

$$(2^5 - 1)^2 - 2$$
$$= 959$$
$$= 7 \times 137$$

which is not prime.

Generator doesn't give 2, 3, 5, 11, …

Centred square primes

$n^2 + (n+1)^2$

Generator gives
5, 13, 41, 61, 113, 181, 313, 421, 613, 761, …
which are prime

Generator also gives

$$(3)^2 + (3+1)^2$$
$$= 25$$
$$= 5 \times 5$$

which is not prime

Generator doesn't give 2, 3, 7, 11, …

Cuban Primes

$$\frac{m^3 - n^3}{m - n}, \quad m = n + 1$$

Generator gives
7, 19, 37, 61, 127, 271, 331, 397, 547, 631, …
which are prime

Generator also gives

$$\frac{6^3 - 5^3}{6 - 5}$$
$$= 91$$
$$= 7 \times 13$$

which is not prime

Generator doesn't give 2, 3, 5, 11, 13, …

Gaussian Primes

$4n+3$

Generator gives
3, 7, 11, 19, 23, 31, 43, 47, 59, 61, …
which are prime

Generator also gives
$4\times3+3$
$=15$
$=3\times5$
which is not prime

Generator doesn't give
2, 5, 13, 17, …

Kynea Primes

$(2^n+1)^2-2$

Generator gives
7, 23, 79, 1087, 66047, 263167, 16785407, 1073807359, …
which are prime

Generator also gives
$(2^4+1)^2-2$
$=287$
$=7\times41$
which is not prime

Generator doesn't give
2, 3, 5, 11, 13, …

Leyland Primes

$n^m+m^n\ ,\ 1<n\le m$

Generator gives
17, 593, 32993, 2097593,8589935681, …
which are prime

Generator also gives
3^4+4^3
$=145$
$=5\times29$
which is not prime

Generator doesn't give
2, 3, 5, 7, 11, …

Pierpoint Primes

2^n3^m+1

Generator gives
2, 3, 5, 7,13, 17, 19, 37, 73, 97, …
which are prime

Generator also gives
$2^3\times3^3+1$
$=217$
$=7\times31$
which is not prime

Generator doesn't give
11, 23, 29, 31, 41, 43, …

Proth Primes

$k\times2^n+1$ with odd k

and $k<2^n$

Generator gives
3, 5, 13, 17, 41, 97, 113, 193, 241, 257, 353, …
which are prime

Generator also gives
$1\times2^3+1$
$=9$
$=3\times3$
which is not prime

Generator doesn't give
7, 11, 19, 23, 29, 31, …

Pythagorean Primes

$4n+1$

Generator gives
5, 13, 17, 29, 37, 41, 53, 61, 73, 89, …
which are prime

Generator also gives
$4\times2+1$
$=9$
$=3\times3$
which is not prime

Generator doesn't give
2, 3, 7, 11, 19, 23, …

Star Primes

$6n(n-1)+1$

Generator gives
13, 37, 73,181, 337, 433, 541, 661, 937,1093, …
which are prime

Generator also gives
$6\times5\times(5-1)+1$
$=121$
$=11\times11$
which is not prime

Generator doesn't give
2, 3, 5, 7, 11, 17, 19, …

Thabit number Primes

$3\times2^n-1$

Generator gives
2, 5, 11, 23, 47, 191, 383, 6143, 786431, …
which are prime

Generator also gives
$3\times2^5-1$
$=95$
$=5\times19$
which is not prime

Generator doesn't give
3, 7, 13, 17, 19, 29, …

Wagstaff Primes

$\frac{2^n+1}{3}$

Generator gives
3, 11, 43, 683, 2731, 43691, 174763, 2796203, 715827883, …
which are prime

Generator also gives
$\frac{2^1+1}{3}$
$=1$
which is not prime

Generator doesn't give
2, 5, 7, 13, 17, 23, 29, …