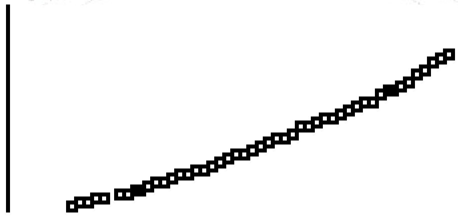
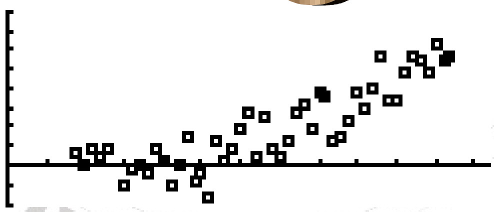
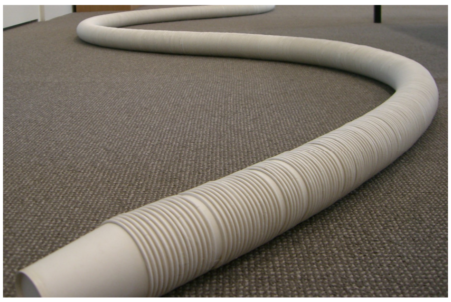


Describing Change





Prince Alfred College
Founded 1869

Noel Baker Centre
for School Mathematics

CASIO®

Describing Change

Version 1.00 – October 2008

Written by Anthony Harradine and Alastair Lupton

Index

Using this resource.

This resource is *not* a text book.

It contains material that is hoped will be covered as a dialogue between students and teacher and/or students and students.

You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan your performance – what questions to ask, when and so on,
- the student that may be absent,
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

Professional development sessions in how to deliver this approach are available.

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Legend.

EAT – Explore And Think.

These provide an opportunity for an insight into an activity from which mathematics will emerge – but don't pre-empt it, just explore and think!

At certain points the learning process should have generated some ***burning mathematical questions*** that should be discussed and pondered, and then answered as you learn more!

Time to Formalise.

These notes document the learning that has occurred to this point, using a degree of formal mathematical language and notation.

Examples.

Illustrations of the mathematics at hand, used to answer questions.

1. *Managing a gas field.*

There is a great deal of planning involved in running a gas production site such as the one pictured.

Once a site has been chosen and gas wells have been drilled, productivity is monitored by measuring the rate of flow of the gas out of each well.

The scenario (a real one)
A gas production site in central Australia contains, potentially, up to six wells. At five of these, wells are already installed and producing gas.

After considering demand levels and production costs, the Reservoir Engineer decides that, for the site to be considered viable, the average daily rate of flow from the entire site in any given month must be at least 5 MMscf/day (millions of cubic feet per day). If the average daily rate for a given month is expected to fall below this, the sixth well will be installed to increase gas production.

The table below gives the actual average daily flow rate from the site for the months shown. During this period only five wells are installed and producing gas.

Month (end date)	Relative time t (months)	Rate of Gas Flow f (MMscf/d)
5/31/1998		51.717
6/30/1998		47.724
7/31/1998		36.717
8/31/1998		31.755
9/30/1998		28.066
10/31/1998		22.248
11/30/1998		22.199
12/31/1998		19.154
1/31/1999		16.377
2/28/1999		14.611
3/31/1999		13.403
4/30/1999		12.72
5/31/1999		11.285

EAT 1

Based on the information provided above, *estimate* the month in which the Reservoir Engineer will require a sixth well in order to increase gas production.

In what ways can you use the data set provided to support your estimate?

12.1

2. *Things that stack.*

Have you ever seen a Cup Snake (sometimes known as a Beer Cup Snake)?

Whilst their preferred habitat is international cricketing fixtures, this photo shows a well-documented sighting at a recent rock concert[1]

EAT 2 A

Obtain 2 disposable cups. By studying *only these two cups*, estimate the length in centimetres (L) of the cup snake made from

100, 200, 300, 400, 500, 600, … n such cups.

Explain how you obtained these estimates.

EAT 2 B

With the help of your classmates, construct a cup snake in such a way as to enable you to check your estimate for the length of a cup snake containing 100, 200, 300, 400, 500 and 600 cups.

Study relationship between your estimated lengths and the measured lengths.
In the light of what you observe, revisit your estimate for the length of a cup snake containing n such cups.

Whilst the length of cup snakes may not be of a great deal of practical significance, a number of objects are stored in stacked form. For those who store such objects, the length or height of such stacks is an important part of this storage.

[1] Foo Fighters concert, Wembley Stadium, June 6th 2008 (the cup snake appeared during the performance of the support act, Australian band Supergrass.).

3. *Talking about change.*

πάντα χωρεῖ καὶ οὐδὲν μένει

Change alone is unchanging

Heraclitus, 500 BC

Change can be observed in most of the systems that make up the world around us, in simple systems like the way the balance of an investment account changes over time, as well as in more complex systems like the growth of plants in response to environmental factors like the supply of water and nutrients.

Human-designed systems, like the investment account, often change in regular and predictable ways, allowing us to model these systems with a high degree of accuracy. Change in non human-designed systems, like the growth of plants, can be somewhat inconsistent. As a result, the modelling of such systems is harder and our ability to predict their behaviour is more questionable.

More specifically

It is a *quantity* associated with a system that changes (i.e. value in $, weight in kg etc). Hence we could say that it is the quantity that shows *variation*.
The variation in one quantity can often be explained by the variation in another.
Consider the following:

To what degree can

...*diamond price* be explained by its *weight*?

...the *incidence of lung cancer* be explained by *degree of exposure to cigarette smoke*?

...*global mean surface temperature* be explained by the *amount of atmospheric carbon dioxide*?

These questions suggest the need to explore the relationship between two *quantities* or *variables*. We can describe this relationship in two ways.

It is likely that, in general,

- variation in *diamond price* is in **response** to variation in its *weight*.
- variation in the *incidence of lung cancer* is in **response** to variation in *exposure to cigarette smoke*.

Or to put that another way, to some degree

- *weight* **explains** the variation in *diamond price*.
- *exposure to cigarette smoke* **explains** the *incidence of lung cancer*.

By thinking in this way we can see that there are two different roles played by the variables in these situations. These roles can be categorised as either

(1) **Response** or (2) **Explanatory**

In a situation, in order to determine which variable plays which role, ask yourself

Which of the variables explains the variation that occurs in the other variable?

> For example, does diamond price explain the variation in weight or does weight explain the variation in diamond price?[2]

In the previous three examples the following is clear,

Response Variable	**Explanatory Variable**
Diamond Price	weight
incidence of lung cancer	degree of exposure to cigarette smoke
global mean surface temperature	amount of atmospheric carbon dioxide

In studies, in laboratories and elsewhere, the idea is to make orderly changes to the *explanatory variable* and observe the variation in the *response variable*.

For example, *nutrient concentration* can be changed in an orderly way (i.e. increased by specific amounts) and the effect on the height of a plant can be observed.

Sometimes, the **response** variable is referred to as the **dependant** variable, as the value it takes tends to depend on the **independent** variable, with is another name for the **explanatory** variable.

We have used the term *explain* not *cause*. The cause of the variation may be something quite different to the explanatory variable.

For example, the price of a new Commodore may be explained by time but what *caused* the variation?

Medical Researchers develop a new drug. They need to find the dose at which it is most effective. The drug is developed to reduce the level of histamines in the body.

What variable are involved and what is their role?

[2] More often than not the response variable is the "interesting variable" and the explanatory variable is the "boring variable".

4. *Polygons and Angles.*

A polygon is a closed figure with n sides. Each and every polygon with a specific number of sides has a set *sum* of its interior angles, as shown in the table below:

number of sides n	3	4	5	6	7	8
angle sum A	180	360	540	720	900	1080

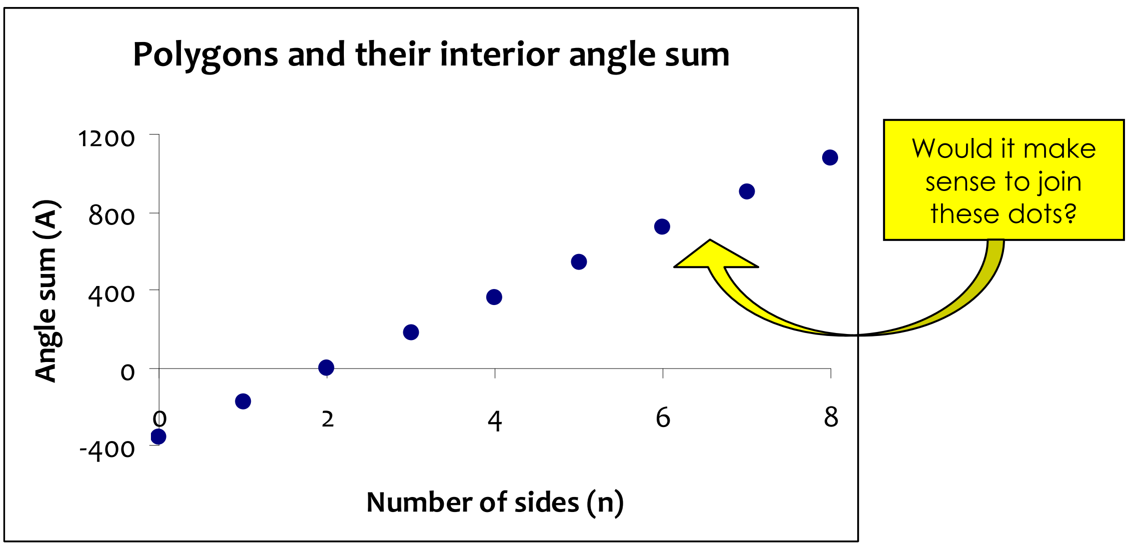
By thinking about how the values of n and A change, this table can be understood in the following way.

consecutive adders in n		+1		+1		+1	
n	3		4		5		6
A	180		180+180		180+180+180		180+180 +180+180
consecutive adders in A		+180		+180		+180	

For a constant *additive* change in the explanatory variable, a constant *additive* change occurs in the response variable.

Change that can be described in this way is referred to as *linear*.

A graphic model of this situation looks as follows:



Note: A useful convention is that the explanatory variable is represented horizontally and the response variable is represented vertically.
This system, the relationship between the interior Angle Sum (A) and the number of sides (n) of a polygon, can be described algebraically.

Members of the family of linear functions[3] $y = mx + c$ can be used to describe systems where a constant *additive* change in the explanatory variable results in a constant *additive* change in the response variable, as seen below

consecutive adders in x	+1	+1	1	$\ldots$	1	
x	0	1	2	3	$\ldots$	x
y	c	$m+c$	$2m+c$	$3m+c$	$\ldots$	$xm+c$ or $mx+c$
consecutive adders in y	m	m	m	$\ldots$	m	

4.1 Can you use the knowledge?

1. Write down the linear function that describes the relationship between the interior Angle Sum (A) and the number of sides (n) of a polygon.

2. a. What is the slope / gradient of the line joining the points on the graph of A against n on the previous page?

   b. What does the slope / gradient represent in the context of system?

3. Explain the meaning of the vertical axis intercept in this context.

4. Use your linear function to determine the angle sum of a dodecagon.

5. Use your linear function to show that no polygon has an angle sum of $1680°$.

6. Through geometric reasoning, it is possible deduce the relationship that we have obtained between the interior angles and the number of sides of a polygon. See if you can do so (a hint is provided in the *Answers* section).

---

[3] A *function* is a mathematical rule that operates on a value (the "input" - often x) to obtain a unique value (the "output" - commonly y). As such, it is a rule for obtaining y by operating on x, sometimes referred to a rule "for y in terms of x".

5. *Modelling Deterministic Change – 1.*

Systems, like the polygon's internal angle sum, that change in a predictable or non-random way are called *deterministic* systems. If a system's change contains a degree of randomness or a limited degree of predictability then it is called a *stochastic* system. We are now going to work with some more deterministic systems.

5.1 Simple Interest.

In non-commercial loans (i.e. amongst family members) *simple interest* is often paid by the borrower to the lender. This is a percentage of the borrowed amount, charged per time period (i.e. per year) of the loan.

Jonathan borrows $5 400 from his father to purchase his first car. To compensate him, Jonathan he agrees to pay him 4% p.a. (per annum or per year) in addition to the borrowed amount (when he can afford to repay the loan).

To study the relationship between the length of the loan (t years) and the amount owed (\$$A$) then

1. Define the role of the two variables.
2. Complete the following table

t	0	1	2	3	4	5
A						

3. Explain why the relationship between these two variables is linear.
4. Represent this relationship graphically.
5. Write down the relationship between A and t algebraically.
(in other words, write down a function of A in terms of t).
6. Use this function to determine the amount that Jonathan owes his father if he repays the loan
   a. after 10 years b. after 7 years and 9 months.
7. How long will the loan have been outstanding at the time when Jonathan owes his father $9 000?

5.2 International Standard (SI) units – Speed.

Speed can be measured in a number of different units. Whilst in science the SI unit *metres per second* is preferred, *kilometres per hour* is more widely understood. Consider the following table,

M (metres per second)	10	20	30	40	50
K (kilometres per hour)	36	72	108	144	180

1. Explain why the relationship between M and K is a linear one.
2. Write down a function for K in terms of M.
3. Use this function to convert
   a. 18.6 m/s to km/h.
   b. 1200 m/s to km/h.
   c. 60 km/h to m/s.
   d. 155 km/h to m/s.

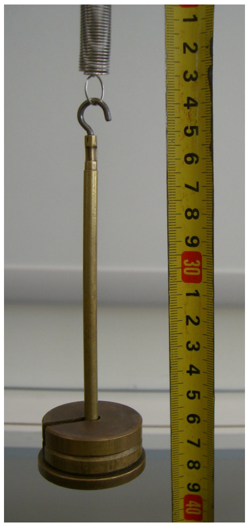
5.3 International Standard (SI) units – Temperature.

Whilst most of the world exclusively use SI units (also known as metric units), USA and Great Britain still frequently work with customary units (a.k.a. imperial units). When obtaining culinary and meteorological information sourced from these countries, temperatures are commonly given in degrees Fahrenheit. Some conversions to degrees Celsius (the SI unit of temperature) are provided below.

F (degrees Fahrenheit)	50	70	90	110
C (degrees Celsius)	10	$21\frac{1}{9}\ \left(=\frac{190}{9}\right)$	$32\frac{2}{9}\ \left(=\frac{290}{9}\right)$	$43\frac{1}{3}\ \left(=\frac{390}{9}\right)$

1. Represent this relationship graphically.
2. What is the 'constant adder' in C that corresponds to a 1 unit increase in F ?
3. Write down an algebraic rule for converting F into C.
4. Use your rule to convert
   a. $95°F$ into $°C$
   b. $360°F$ into $°C$
5. Develop an algebraic rule for converting C into F.
6. Hence or otherwise convert
   a. $0°C$ into $°F$
   b. $40°C$ into $°F$

6. *Signal and Noise.*



Recall the statement made earlier

> In studies, in laboratories and elsewhere, the idea is to make orderly changes to the *explanatory variable* and observe the variation in the *response variable*.

It is time to look at the insights into change that can be had when this approach is applied to a simple system like the one pictured.

EAT 3

Set up an experiment that will allow you to gather information about the way that the length of a helical spring (hung at one end from a fixed point as illustrated) changes when a range of weights are suspended from the other end.

Describe the variables that will be studied in your experiment and the roles that they play.

Predict the results of your experiment.

Conduct the experiment, record the results and compare with your predictions.

6.1 Modelling stochastic change.

1. Complete a table similar to the following with the data that you obtained from your experiment.

consecutive adders in m						
w (weight in grams)						
l (spring length in cm)						
consecutive adders in l						

[4]

For the constant additive change that you affected in the explanatory variable, the change in the response variable was probably not *exactly* a constant additive one, but *sort of* …

What does this suggest about a graphical representation of the relationship between spring length and weight?

2. Represent this relationship graphically.

From your graph, two things should be clear.

- This system is not completely deterministic; there is some unpredictability in the change that we are studying.
- The relationship between the variables is roughly linear.

As a consequence of these realisations, two more things should be apparent

- We can *approximate* the relationship between m and l with a linear function.
- Such a function will represent the general trend but not the unpredictable change, and so any use of such a model will not take into account this behaviour.

This leaves us with two questions

- what linear function would best "fit" this relationship, and
- how to find it?

[4] If you cannot complete the experiment, use the results obtained by the authors,

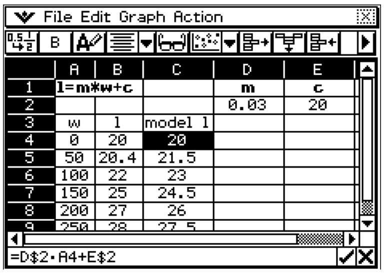
w (weight in grams)	0	50	100	150	200	250
l (spring length in cm)	20	20.4	22	25	27	28

6.2 Seeking a line of best fit

1. Using your choice of electronic technology, enter your data into a spreadsheet.

2. In two cells (e.g. D2 and E2), enter an estimate for the gradient/slope and l-intercept of your linear model for l in terms of w.

3. Use these values within a formula that will calculate your model's "predicted" l values for the w values used in your experiment (see Column C)[5]



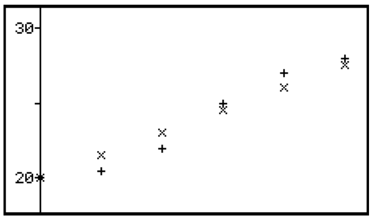
	A	B	C	D	E
1	l=m*w+c			m	c
2				0.03	20
3	w	l	model l		
4	0	20	20		
5	50	20.4	21.5		
6	100	22	23		
7	150	25	24.5		
8	200	27	26		
9	250	28	27.5		

4. Adjust your estimates for the gradient/slope and l-intercept of your linear model until you think you have found the equation of your "line of best fit".

5. Compare the equation of your "line of best fit" with that of your classmates.

But what is the BEST line of best fit?

- One way to start is to average the measured consecutive adders in l to estimate your constant adder (i.e. gradient/slope).

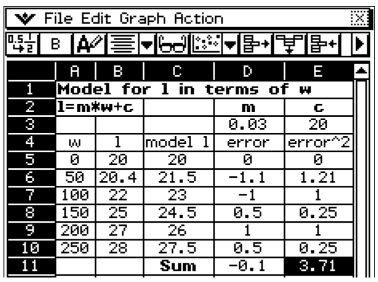


- A graph of w against model l and actual l is *very* helpful.

- Characteristic of a good line of best fit is that, generally, there will be roughly as many points where model l is greater that actual l (i.e. above on graph) as there are points where model l is less that actual l (i.e. below on graph). These 'gaps' should be as small as possible and should be as random as possible in their distribution.

- The BEST line of best fit is one that *minimises the sum of the squares of the errors*, where the errors are the differences between the model l and actual l for each w value.

- Your spreadsheet can be added to, to measure errors and the sum of their squares.



	A	B	C	D	E
1	Model for l in terms of w				
2	l=m*w+c			m	c
3				0.03	20
4	w	l	model l	error	error^2
5	0	20	20	0	0
6	50	20.4	21.5	-1.1	1.21
7	100	22	23	-1	1
8	150	25	24.5	0.5	0.25
9	200	27	26	1	1
10	250	28	27.5	0.5	0.25
11			Sum	-0.1	3.71

[5] See Section **12.2** for support in using a CASIO ClassPad to build this spreadsheet.

6.3 Using a stochastic model.

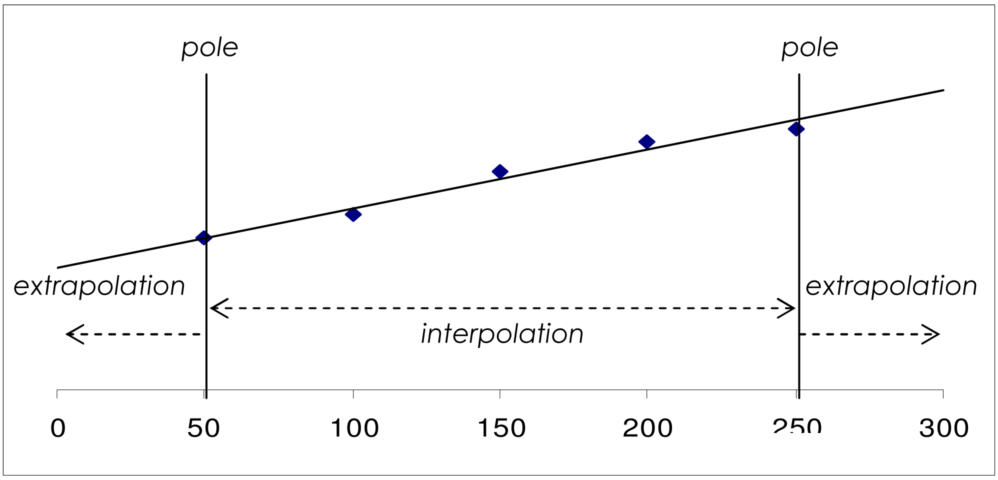
Once the line of best fit is determined and you have its equation then you can use it to *predict* other values that the experiment did not reveal.

Discuss why have the word *predict* is used in this context.

Interpolation and Extrapolation.

Using your model to predict length values for mass values *in between the smallest and largest masses* in the experimental data (the "poles" of the data) is called *interpolating* ("within the poles").

Using your model to predict length values for mass values outside of the smallest and largest masses in the experimental data is called *extrapolating* ("beyond the poles").



The accuracy of an interpolation depends on one feature of the original data.

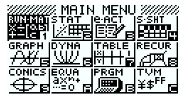
1. What is this feature??

The accuracy of an extrapolation depends on this same feature of the original data and a rather large assumption in most cases.

2. What is the rather large assumption?

3. Discuss the validity of this assumption in the case of the spring experiment.

7. *Modelling Stochastic Change – 1.*



12.3

7.1 Atmospheric Carbon Dioxide (CO_2) over time – part 1.

For many years the concentration of Carbon Dioxide in the atmosphere (measured in p.p.m. – parts per million) has been recorded at Mauna Loa, Hawaii. The most recent 8 years of data from Mauna Loa are recorded below.

Year	2000	2001	2002	2003	2004	2005	2006	2007
Atmospheric concentration of Carbon Dioxide (p.p.m.)	369.48	371.02	373.1	375.64	377.38	380.99	382.71	384.5

1. Represent this data graphically.
2. By considering the average additive change in C, the quantity of atmospheric CO_2 in p.p.m., over successive years, write down a linear model for C in terms of t, the number of years since 2000.
3. Sketch your model from part 2 on your graph from part 1.
4. Refine, if necessary, the co-efficients of your linear model in the light on this sketch.
5. Use your linear model to predict the concentration of Carbon Dioxide in the atmosphere in the year 2010.
6. Scientist warn of the potential significant climate change if the concentration of Carbon Dioxide in the atmosphere exceeds 400 p.p.m. According to your model, in what year will that quantity be exceeded?
7. What assumption are you making in your answering on part 6?

7.2 A stochastic process.

A stochastic process has generated the following data:

x	5	10	15	20	25	30	35	40	45	50
y	997	932	893	825	837	797	727	646	605	611

1. Draw a scatter plot of this data.
2. Calculate the average additive change in y for a one-unit increase in x, and hence develop a linear model for y in terms of x.
3. Sketch your model from part 2 on your graph from part 1.
4. Refine, if necessary, the co-efficients of your linear model in the light on this sketch.
5. Use your linear model to predict the result of this stochastic process if $x = 65$.

7.3 AFL – team success and individual acclaim.

At the end of the 2008 AFL Australian Rules Football season data was gathered on each team including: its ladder position, the number of games won and the total number of Brownlow Medal (fairest and most brilliant player in the AFL) votes awarded to its players. This data was as follows:

Team	Ladder Position	No. of games won	Total Brownlow Votes
Geelong	1	21	112
Hawthorn	2	17	91
Western Bulldogs	3	15.5	83
St Kilda	4	13	75
Adelaide	5	13	78
Sydney	6	12.5	78
North Melbourne	7	12.5	58
Collingwood	8	12	65
Richmond	9	11.5	69
Lions	10	10	66
Carlton	11	10	68
Essendon	12	8	45
Port Adelaide	13	7	57
Fremantle	14	6	56
West Coast	15	4	31
Melbourne	16	3	24

1. Represent the relationship between the ladder position and total number of Brownlow Medal votes for the AFL teams in 2008 graphically.
2. Define two variables, and their roles, in relation to this scatter plot.
3. Calculate the average additive change in your response variable for a 1-unit increase in your explanatory variable.
4. Hence develop a linear model of best fit for the relationship between the ladder position and total number of Brownlow Medal votes for the AFL teams in 2008.
5. Express the average additive change from your linear model in a sentence that would be understandable to an AFL supporter.
6. Repeat parts 1 to 5 in relation to the number of games won and total number of Brownlow Medal votes gained.
7. Which of the two explanatory variables used above is more appropriate to explain the variation in the total number of Brownlow Medal votes gained by an AFL team in 2008. Give a reason for your answer.

7.4 The Price of Diamonds.

To investigate exactly how the price of a diamond relates to its weight, a random sample of 40 loose diamonds for sale through an online gem dealer was taken. All were "round cut" and were classified as having clarity "VSI" (very slight inclusions) and colour D to H (near colourless).

The weight (w in carats) and price (p in \$) of these diamonds is recorded in the following table:

w	0.27	0.27	0.28	0.25	0.31	0.35	0.36	0.32	0.32	0.36
p	509	509	518	582	597	660	661	670	670	678

w	0.3	0.36	0.33	0.28	0.31	0.31	0.3	0.33	0.33	0.3
p	696	700	727	747	782	782	800	808	829	849

w	0.37	0.32	0.32	0.39	0.39	0.41	0.4	0.4	0.41	0.39
p	851	852	859	890	890	981	1017	1068	1071	1113

w	0.39	0.42	0.42	0.42	0.41	0.43	0.42	0.46	0.46	0.48
p	1113	1119	1310	1331	1398	1476	1516	1543	1595	1803

1. Draw a scatter plot of p against w.
2. Draw on this scatter plot the line of best fit (determined 'by eye').
3. By obtaining accurate information from the line of best fit that you have drawn, determine the equation of your line of best fit.
4. Provide an interpretation for the slope/gradient of your equation.
5. Use your model to predict
    a. The cost of a diamond weighing 0.45 carats
    b. The cost of a diamond weighting 0.55 carats.
    c. The size of diamond likely to cost \$2000.

7.5 Unleaded Petroleum and the price of Crude Oil.

An economics teacher mentions to their class that a one dollar increase in the barrel price of Crude Oil adds one cent per litre to the price of unleaded petrol.

To investigate the accuracy of this "rule of thumb" the following data has been collected:

2006

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
Crude Oil (\$US/barrel)	58.3	54.65	55.42	62.5	62.94	62.85	66.28	64.93	55.73	50.98	50.98	54.06
ULP (cents/litre)	117.7	117.9	120.6	128.6	135.6	135.5	135	133.3	122.3	116.3	112.7	115.7

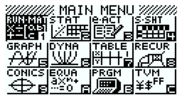
2007

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
Crude Oil (\$US/barrel)	46.53	51.36	52.64	56.08	55.43	59.25	65.96	64.23	70.94	77.56	86.92	83.46
ULP (cents/litre)	115.2	114.9	122.5	124.8	130.2	129.9	126.7	122.7	123.8	124.3	130.5	137.1

2008

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*
Crude Oil ($US/barrel)	84.7	86.64	96.87	104.31	117.4	126.33
ULP (cents/litre)	138.6	136.1	139.7	143.1	148.9	157.8

1. Define the variables and their roles in this situation.
2. Develop a model that best fits this data.
3. With reference to your model, comment on the "rule of thumb" quoted above. If necessary, formulate a revised rule of thumb.
4. Use your model to predict
   a. The price of ULP if crude oil cost $80 per barrel.
   b. The level to which the cost of crude oil will have risen if the cost of ULP reaches $8.00 per litre (*CSIRO worst case scenario for 2018*).



12.4

8. Invest smarter.

Compound Interest … explains why $1000 … can grow to $47 000 over 50 years. No wonder Albert Einstein described compound interest as one of the greatest human discoveries

ipac finacial services

EAT 4 How does Compound Interest work?

Consider an investment of $1 000 that earns 6% interest p.a.

In the 1st year, how much interest is earned?

Therefore, what is the total value of the investment after the 1st year?

Based on this,

In the 2nd year, how much interest is earned?

Therefore, what is the value of the investment after the 2nd year?

Repeat this for the 3rd, 4th and 5th years.

Can you write down a model for V, the value of the investment after n years?

Is this a deterministic or stochastic model? Why?

If you can write down a model for V in terms of n, use it to predict the value of the investment after 50 years.

If this value is not $47 000, how would the scenario have to changed if the claim above was to be accurate?

What different ways were used to perform the calculations required in **EAT 4**?

Which way was *easiest*?

How do you (easily) increase an amount by 15%? 80%? 1%?

Building a model for compound interest.

In **EAT 4** you should have performed the equivalent to the following calculations

n	V (Value in \$ at end of n th year)
0	1000
1	$1000 \times 1.06 = 1060$
2	$1060 \times 1.06 = 1123.60$
3	$1123.60 \times 1.06 = 1191.02$
4	$1191.02 \times 1.06 = 1262.48$
5	$1262.48 \times 1.06 = 1338.23$

This process of repeated multiplication by 1.06 can be examined further

n	V	V	V
0	1000	1000	1000×1.06^0
1	$1000 \times 1.06 = 1060$	$(1000) \times 1.06$	1000×1.06^1
2	$1060 \times 1.06 = 1123.60$	$(1000 \times 1.06) \times 1.06$	1000×1.06^2
3	$1123.60 \times 1.06 = 1191.02$	$(1000 \times 1.06 \times 1.06) \times 1.06$	1000×1.06^3
4	$1191.02 \times 1.06 = 1262.48$	$(1000 \times 1.06 \times 1.06 \times 1.06) \times 1.06$	1000×1.06^4
5	$1262.48 \times 1.06 = 1338.23$	$(1000 \times 1.06 \times 1.06 \times 1.06 \times 1.06) \times 1.06$	1000×1.06^5

From this table it should be clear that the deterministic model for V in terms of n is

$$V = 1000(1.06)^n$$

The initial value of the investment

The constant multiplier per unit increase in n

This is an example of a *simple exponential function*

$$y = a \times b^x$$

y value when x=0 (y- intercept)

The constant multiplier per unit increase in x

This has a direct parallel with the linear function $y = mx + c$ where c and m represent the y-intercept and the constant adder respectively.

8.1 Can you use the knowledge?

1. Explain why a constant multiplier of 1.06 implies that the previous value is increased by 6%.

2. What sort of change would be created by a constant multiplier of
   a. 1.45?
   b. 1.025?
   c. 0.95?
   d. 0.825?

3. Write down a function that would model V, the value of an investment of
   a. \$6250 invested at 8.5% compound interest for n years.
   b. \$14.92 invested at 2.25% compound interest for n years.
   c. \$115 000 invested at 11% compound interest for n years.

9. *Simple exponential functions.*

EAT 5 Investigating the graph of $y = a \times b^x$.

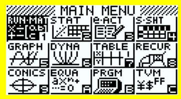
Select a fixed value for a.

Choose a range of values for b (what sort of numbers could you use?)

Using your a value, and a different b value for each, draw 4 or 5 sketches of $y = a \times b^x$ on the same large set of axes.

Summarise your findings with respect to the effect that different b values have on the graph of $y = a \times b^x$.

Repeat this investigation with a fixed b value and a range of values for a.



12.5

Glass - thickness and transparency.

A glass manufacturer plans to produce a type of glass where its transparency (the amount of light that passes through it) depends on its thickness.
According to glass making theory the relationship will be as follows,

consecutive adders in x		$+1$		$+1$		$+1$	
Glass Thickness x (mm)	0		1		2		3
Transparency T (lumens)	100		80		64		51.2
consecutive multiplier in T		$\times 0.8$		$\times 0.8$		$\times 0.8$	

Like compound interest calculations, this deterministic relationship between glass thickness and transparency exhibits the following property,

For a constant *additive* change in the explanatory variable, a constant *multiplicative* change occurs in the response variable.

This is sometimes referred to as *exponential* or *multiplicative* change.

In such a case, where a constant multiplier (i.e. 0.8) is evident and an initial value is provided, a simple exponential model is appropriate and easy to obtain i.e.

$$T = 100 \times 0.8^x$$

Initial transparency

The constant multiplier

 Draft, Oct 2008 WIP

10. Modelling Deterministic Change – 2.

"Based on the data below, predict the x value when $y = 900$"

x	3	6	9	12	15	18	21
y	7	12	21	36	62	107	184

Finding a simple exponential model.

Determining the average multiplicative change in the y values above

$$(1.714 + 1.75 + 1.714 + 1.722 + 1.726 + 1.720) \div 6 = 1.724$$

We know that, if $y = a \times b^x$ then $b \times b \times b = b^3 = 1.724$

As the multiplicative change of 1.724 is caused by a <u>3-unit increase</u> in x *(i.e.by three applications of the constant multiplier).*

Hence $b = \sqrt[3]{1.724} = 1.20$.

Further $a = 7 \div 1.20 \div 1.20 \div 1.20 = 7 \div 1.724 = 4.06$

This is working backwards to the "initial value" – the value of y when $x = 0$.

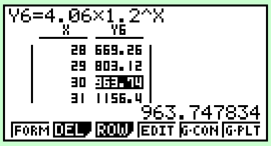
So our model for y in terms of x is $y = 4.06 \times 1.2^x$

Using a simple exponential model.

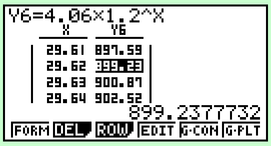
Our question can now be answered by solving $4.06 \times 1.2^x = 900$, … but how …?

Method 1 – Tabular

A rough answer



which can be refined

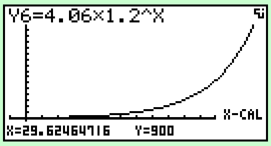


Method 2 – Graphical

Sketch the model.

Get a `View Window` containing the solution.

Perform an `X-cal` `G-Solv` **[F5]** and **[F6]** then `X-Cal` **[F2]**.



Method 3 – Algebraic

$4.06 \times 1.2^x = 900$

$\therefore 1.2^x = 221.675$

$\therefore x = \log_{1.2} 221.675$

$\therefore x \approx 29.625$

(*calc's to 3 dec. places*)

10.1 Seismic Energy – the Richter Scale

The Richter scale represents the amount of seismic energy released during an earthquake or other event. Richter magnitudes can be equated to other units of energy like mega joules (MJ).

The table below provides the mega joule equivalent to six Richter magnitudes

consecutive adders in R						
R (Richter magnitude)	0	1	2	3	4	5
M (energy in megjoules)	4.2	132.8	4200	132816	4200000	132815662
consecutive multiplier M						

1. Using the previous table, write down a model for M in terms of R.

2. Use this model to determine the energy released (in MJ) by the earthquake that devastated Kashmir in 2005 which measured 7.5 on the Richter Scale.

3. The Valdivia Earthquake that struck Chile in 1960 measured 9.5 on the Richter Scale, one of the highest ever recorded.
   How many MJ of energy did it release?

4. Describe the difference in energy release between earthquakes that differ by two on the Richter scale.

10.2 Depreciation.

According to a company's *Depreciation Schedule*, an assets "book value" in the years after purchase is as follows;

t (years since purchase)	0	1	2	3	4	5
V (book value – whole $'s)	112500	95625	81281	69089	58725	49916

1. Interpret the constant multiplier evident in the relationship between V and t.

2. Write down a simple exponential model for this relationship.

3. Hence determine the book value of the asset
   a. 10 years after purchase.
   b. 13.5 years after purchase.

4. In what year does the book value of the asset fall below $5000?

10.3 Radioactive Decay.

The radioactive isotope Thorium 234 decays (by beta-emission of radioactivity into protactinium-234) in the following way

t (days)	0	4	8	12	16	20
W (mass of remaining Thorium-234 in grams)	100	89	79.4	70.7	63	56.1

1. Determine the constant multiplier in W for a one-unit increase in t.

Interpret this value in the context of the question.

2. Write down a simple exponential model for this relationship.

3. Hence the amount of Thorium-234 remaining after
   a. 7 days.
   b. 365 days.

4. What is the half-life of Thorium-234?
   (An isotope's half-life is the time taken for it to decay to half its mass)

10.4 Sound Intensity and Loudness.

Sound intensity, a measure of the energy travelling in sound waves, is measured in *watts per m²*. Sound loudness is measured in *decibels*.
The two are related in the following way

d (db - decibels)	0	10	20	30	40	50
I (watts per m²)	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}

1. Determine the constant multiplier in I for a one-unit increase in d .
2. Write down a simple exponential model for this relationship.
3. Hence determine the Intensity, in watts per m², of loud with loudness of
   a. 56 db.
   b. 105 db.

10.5 More on compound interest.

Investments and loans are commonly compounded more often than annually.
If so then the quoted interest rate is divided by the number of compounds per year, and the (new and smaller) constant multiplier is "applied" once per compounding period. For example, if our $1000 was to earn 6% compounded quarterly (4 times per year) then our model for its value would become

$V = 1000(1.015)^n$ where n is now the length of the investment in *quarters*.

1. Determine the value of our $1000 investment after 50 years of earning 6% compounded quarterly.
2. Write down a model for V if the investment were compounded monthly.
3. Hence, determine its value after 50 years in this situation.
4. Repeat part 2 and 3 with interest that is compounded daily.

Taking compounding to the limit …

5. Consider $1 invested at 100% p.a. (a theoretical investment obviously).
   Calculate the value of this "investment" after 1 year if the interest was compounded with increasing frequency i.e.

Compounding period	quarterly	monthly	daily	every second	continuously
Value of investment					

11. Modelling Stochastic Change – 2.

11.1 Managing a Gas Field – 2

Revisit the *Managing a Gas Field* scenario on page 5.

Recall your prediction about the month in which the 6^{th} gas well was to be required.

Now we can use our increased knowledge of algebraic models to refine this prediction.

1. If you have not already, draw a scatter plot of the gas flow data on page 5.
2. What property of the data (or of the context of gas extraction) suggests that a simple exponential function could model the relationship between flow and time?
3. Investigate an approximate constant multiplier in the change in flow.
4. Hence formulate an algebraic model for the relationship between flow and time.
5. Use your model to predict when the 6^{th} well will be required.
6. Compare this with your earlier prediction. How did you do?

As it happens, more data is available from this gas field as follows

Month (end date)	Relative time t (months)	Rate of Gas Flow f (MMscf/d)
6/30/1999		12.992
7/31/1999		9.21
8/31/1999		8.836
9/30/1999		5.874
10/31/1999		4.938
11/30/1999		11.775
12/31/1999		16.709
1/31/2000		15.579
2/29/2000		14.861
3/31/2000		14.067
4/30/2000		26.285
5/31/2000		28.882
6/30/2000		24.963
7/31/2000		23.124
8/31/2000		20.43
9/30/2000		18.963
10/31/2000		17.335
11/30/2000		15.61
12/31/2000		14.516

7. In what month was the 6^{th} well required? How good was your prediction?
8. Does it seem as if the 6^{th} well was ready at that time?
9. Using the additional data, develop a prediction for when the flow of gas from this field will next fall below 5 MMscf/d, making the field unviable.

11.2 Bouncing ball

A scientist dropped a steel ball from a height of 10 metres onto a very hard surface, and allowed it to bounce several times, yielding the following data,

Bounce number (n)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Height after bounce (H)	10	8.6	7.4	6.0	5.1	4.5	4.0	3.2	2.8	2.2	2.0	1.8	1.5	1.2	1.0
Consecutive adders in n															
Consecutive multipliers in H															

1. Fill in the last two rows of this table.
2. Draw a scatter plot of H against n.
3. Use the values in the table to write down a simple exponent model for H in terms of n.
4. Represent this model on your scatter plot.
5. Explain why, in this context, interpolation is of little relevance.
6. Using your model, predict the height of the 20th bounce.
7. Using your model, predict which will be the first bounce with a height of less than 10 centimetres.

11.3 Another stochastic process.

A stochastic process has generated the following data:

x	10	11	12	13	14	15	16	17	18	19
y	103.2	101.9	105.1	111.4	111.7	118.5	115.4	121.6	126.7	129.7

1. Draw a scatter plot of this data.
2. Calculate the average multiplicative change in y for a one-unit increase in x.
3. Use your answer to part 2 to develop an approximate value for y when $x = 0$.

   and hence develop a simple exponential model for y in terms of x.
4. Sketch your model from part 3 on your graph from part 1.
5. Refine, if necessary, the co-efficients of your simple exponential model in the light on this sketch.
6. Use your linear model to predict the result of this stochastic process if $x = 25$.

11.4 Worldwide use of the WWW.

According to an internet monitor, the percentage of the world's population that uses the internet (at least monthly) has changed, since 1995, in the following way:

years since '95	0	1	2	3	4	5	6	7	8	9	10	11	12
WWW use (%)	0.4	0.9	1.7	3.6	4.1	5.8	8.6	9.4	11.1	12.7	15.7	16.7	20

1. Represent this data on a scatter plot.
2. Explain why a linear model is not able to capture the change in use of the world wide web since 1995.
3. Investigate the possibility of modelling this change with a simple exponential model.
4. Use technology to obtain a quadratic model of best fit for this data.
5. Use your preferred model from parts 3 or 4 to predict when a third of the world will use the internet (at least monthly).
6. How accurate do you think the prediction asked for in part 5 is likely to be?

11.5 Global Temperature and Atmospheric CO_2.

The table provided contains the annual data from Mauna Loa, the longest running atmospheric monitoring station based in Hawaii, along with the global mean land-ocean temperature (relative to the 1951-1980 mean) for the years after 1959.

Investigate appropriate models for

1. atmospheric CO_2 in terms of years since 1959,
2. global mean land-ocean temperature in terms of years since 1959
3. global mean land-ocean temperature in terms of atmospheric CO_2.

In each case,

a. Assign variables and classify their role.

b. Determine the type of algebraic model that you will use to describe the relationship between the variables, explaining the reasons for your selection.

c. Determine the equation of the line / curve of best fit, using the concepts of "constant adder / multiplier" and a "graph and refine" method, with documentation.

d. Use the model fitting capacity of your choice of technology to find the equation of an alternative line / curve of best fit.

e. Compare the models obtained in parts c and d and select your choice of model. Give reasons for your selection.

f. Use your choice of model to make predictions about 2020.

g. Use technology to obtain a quadratic model of best fit (if appropriate).

h. Compare this model with your choice in part e in terms of
 i. fit to data
 ii. predictions about 2020.

i. Discuss the limitations inherent in your results.

year	Global atmospheric concentrations of carbon dioxide	global mean land-ocean temperature (relative to 1951-1980 mean)
1959	315.83	0.06
1960	316.75	-0.01
1961	317.49	0.08
1962	318.3	0.04
1963	318.83	0.08
1965	319.87	-0.11
1966	321.21	-0.03
1967	322.02	0
1968	322.89	-0.04
1969	324.46	0.08
1970	325.52	0.03
1971	326.16	-0.1
1972	327.29	0
1973	329.51	0.14
1974	330.08	-0.08
1975	330.99	-0.05
1976	331.98	-0.16
1977	333.73	0.13
1978	335.34	0.02
1979	336.68	0.09
1980	338.52	0.18
1981	339.76	0.27
1982	340.96	0.05
1983	342.61	0.26

year	Global atmospheric concentrations of carbon dioxide	global mean land-ocean temperature (relative to 1951-1980 mean)
1984	344.25	0.09
1985	345.73	0.05
1986	346.97	0.13
1987	348.75	0.27
1988	351.31	0.31
1989	352.75	0.19
1990	354.04	0.38
1991	355.48	0.35
1992	356.29	0.12
1993	356.99	0.14
1994	358.88	0.24
1995	360.9	0.38
1996	362.57	0.3
1997	363.76	0.4
1998	366.63	0.57
1999	368.31	0.33
2000	369.48	0.33
2001	371.02	0.48
2002	373.1	0.56
2003	375.64	0.55
2004	377.38	0.49
2005	380.99	0.62
2006	382.71	0.54
2007	384.5	0.57

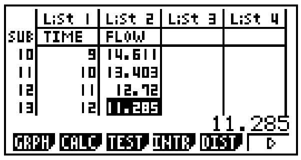
NB: A good way to demonstrate the skills that you have learned in this unit would be to complete Section 11.5 in the form of a mathematical report. To do this:

- Link the subsections with sentences or paragraphs that show that you know what you are doing and why you are doing it as well as what your results mean (this should mean you will not need to refer to part a, b etc).
- Look for ways to extend your work beyond the minimum requirements.

12. *eTech Support.*

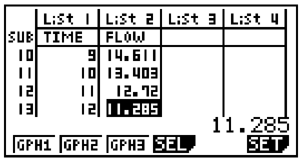
12.1 Representing bi-variate data – the Scatter Plot.

Enter STAT in your CASIO *9860G AU*.



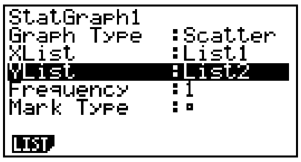
Enter the data set into two Lists (i.e. `List 1` and `List 2`). You can name these lists by placing the input bar in the `sub` row and typing letters using the red alpha keys (i.e. ALPHA ÷ to obtain `T`).

Enter the graph sub-menu by pressing `GRPH` F1, and then check and/or adjust the graph set up by pressing `SET` F6.

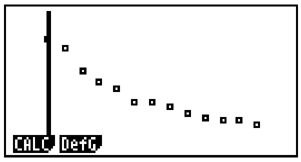


In the set up window

- the `Graph Type` can be set as a Scatter Plot by moving the input bar down and pressing `Scat` F1.
- the `XList` and `YList` can be set as the location of your data. Move the input bar down and press `List` F1, followed by the list number i.e. 1.
- If desired, the `Mark Type` can be altered in a similar way.



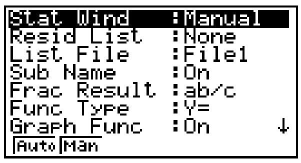
With the graph set up correctly, press EXE to exit the set up window and then press `GPH1` F1 to draw the Scatter Plot.



Manipulating a Scatter Plot – the View Window.

In STAT the `View Window` automatically adjusts to include your Scatter Plot.

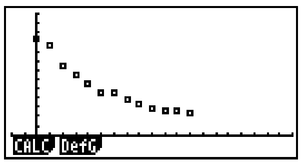
This can be over-ridden by entering the `SET UP` by pressing SHIFT and MENU, and then changing the `Stat Wind` to `Manual` by pressing `Man` F2.



Press EXE or EXIT to exit the `SET UP` and then re-draw the Scatter Plot.

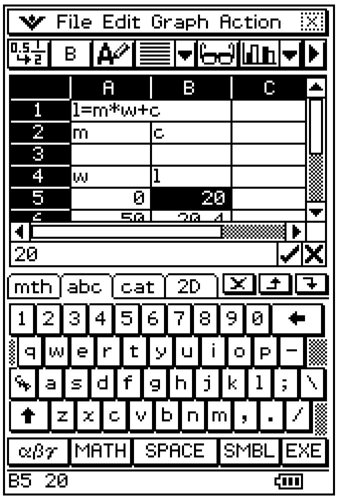
The graph's `View Window` can now be altered, (to help us consider how the gas might flow in the future) by pressing SHIFT and `V-Win` F3.

By moving the input bar down and setting the values (to the ones shown) we can re-draw the graph on visible, extended, clearly scaled axes. This may help us to estimate what future flow values might be, by following the observable trend 'by eye'.



12.2 Model fitting with a spread sheet (using the CASIO ClassPad 300).

The process of seeking a line of best fit, in the way described below, can be performed on a range of spreadsheet packages including the CASIO ClassPad 300, the CASIO 9860G AU and Microsoft Excel. The instructions below relate to the CASIO ClassPad 300 but could be adapted for other packages.



Enter the spreadsheet mode of a CASIO ClassPad.

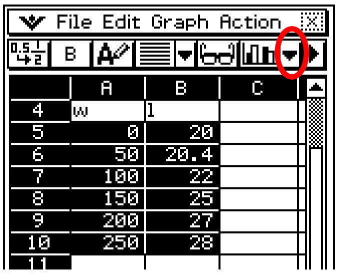
Text can used to label aspects of a spreadsheet.

To do so, tap on the cell and enter text using the Keyboard.

In this case the labels in Row 2 relate to the slope (m) and y-intercept (c) that will be later stored in Row 3.

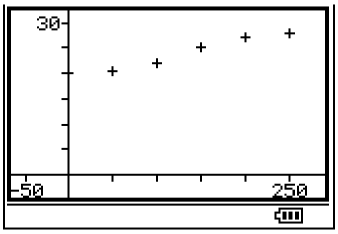
The numerical data can be entered in a similar way (into Column A and Column B say).

Graphing Spreadsheet data.



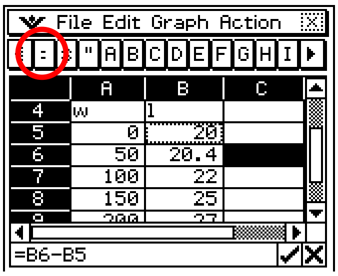
To graph data that is in a spreadsheet, first select the data, then choose a type of graph from the selection obtained by tapping the drop-down arrow next to the graph icon (circled top right).

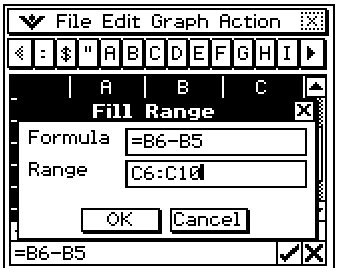
Tap to obtain a Scatter Plot of the selected data.



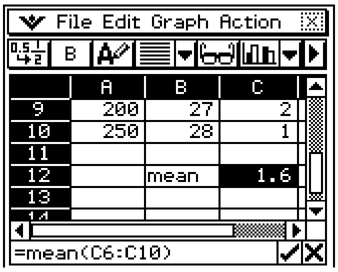
Investigating constant adders.

The consecutive adders in the l-values can be found by entering the formula =B6-B5 into Cell C6. This formula can be filled down Column C by tapping Edit : Fill Range. (with cell C6 selected) and setting the Range as C6:C10. (the : is available from top left of the screen as circled below)





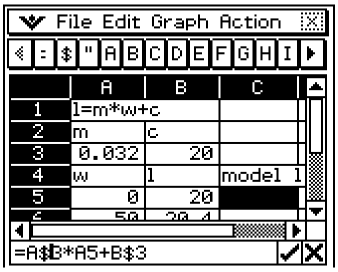
The mean ("average") consecutive adder can be calculated by entering the formula `=mean(C6:C10)` (into Cell `C12` say).



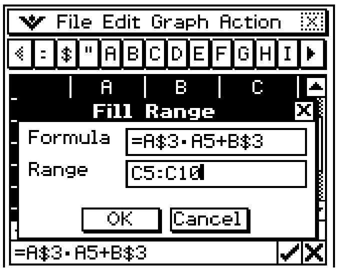
Based on this, a 'first guess' for the slope of the linear model would be $m = \frac{1.6}{50} = 0.032$

By combining this with the initial `l`-value of 20 as a 'first guess' for the vertical axis intercept, we are ready to seek our line of best fit by storing these values in cells `A3` and `B3` respectively.

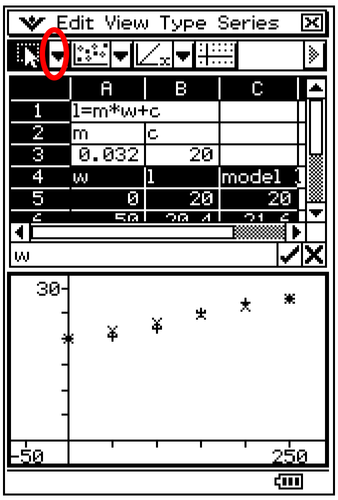
Seeking a line of best fit.



Because, in this spreadsheet `m=A$3`, `w=A5… A10` and `c=B$3`,the equation of the line of best fit, $l = m \times w + c$ becomes the formula `=A$3*A5+B$3`, to be entered into `C5` and then filled down using `Edit : Fill Range`.

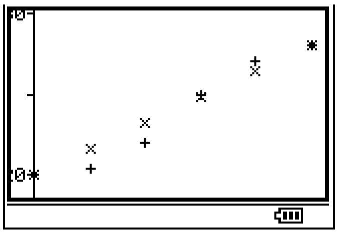


Note: the $ is necessary so that, when the formula is filled down, the reference to the values in row 3 does not 'move down' to row 4, 5, … (this 'moving down' is desirable for the `w` values in Column `A` but is not desirable for the `m` and `c` values).



We can evaluate the degree of 'fit' of this model in a number of ways.
We could graph both the measured `l` vales and the model's `l`-values against `w`.

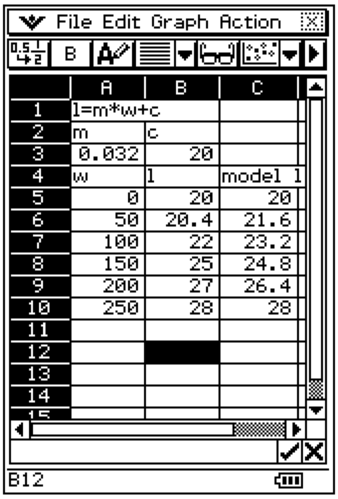
This can be made clearer by 'zooming in' by selecting `Zoom Box` from the drop down menu (next to the Arrow icon circled top right) and then drawing a box over the relevant part of the graph.



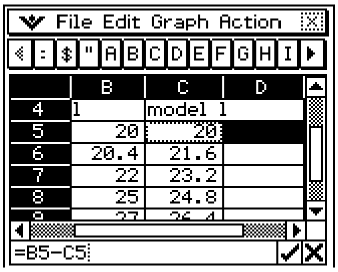
This graph more clearly shows that the model (×'s) 'over-predict' `l` for small `w` values and under-predict `l` for larger values of `w`.

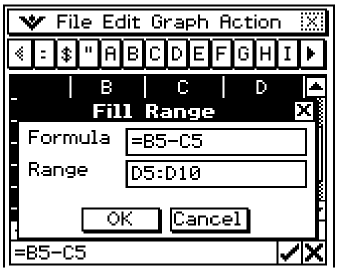
Measuring Error

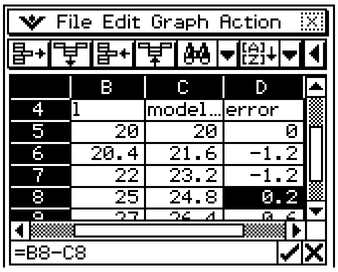
Another way to evaluate the degree of 'fit' of our model is to compare the predicted and measured `l`-values numerically. This can be done by 'eye-balling' the values in columns `B` and `C`.



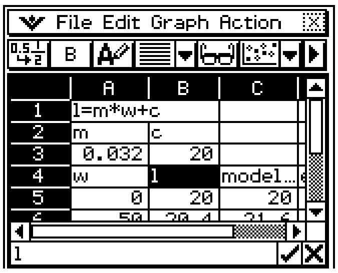
A way to quantity this 'fit' is to measure the 'error', that is, the difference between the measured values and the predicted values i.e measured `l` – predicted `l`.
In our spreadsheet that means using the formula
`= B5 - C5` and filling it down over `D5:D10`.
This process can be seen below.

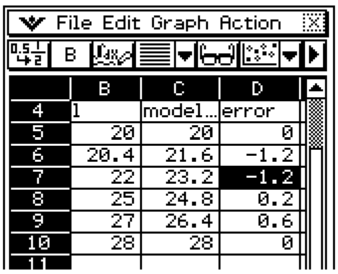


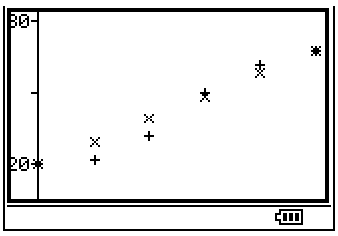


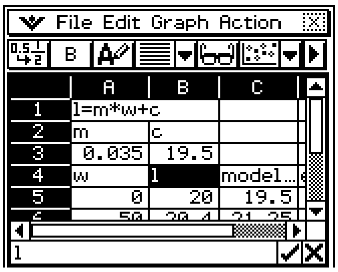


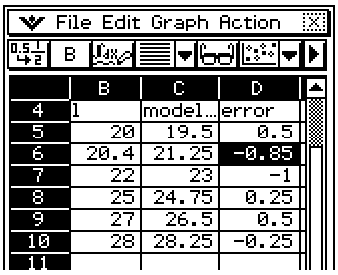
Now that it is built this, spreadsheet allows you to modify the `m` and `c`-values of the linear model in order to improve its fit. Below you can see how the initial values compare to a pair of adjusted `m` and `c`-values

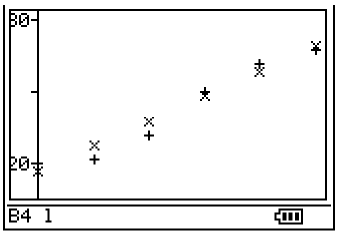






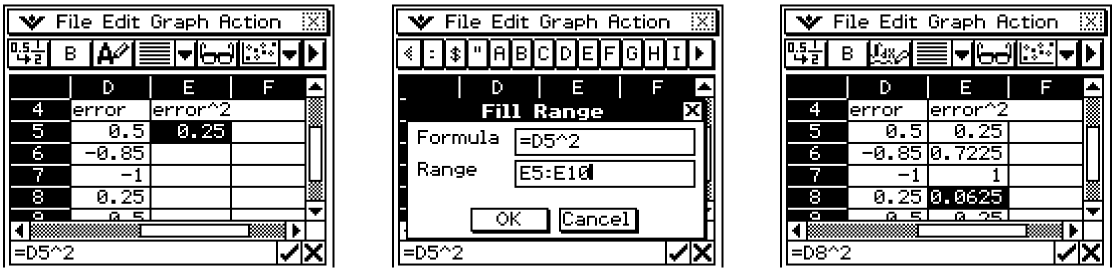




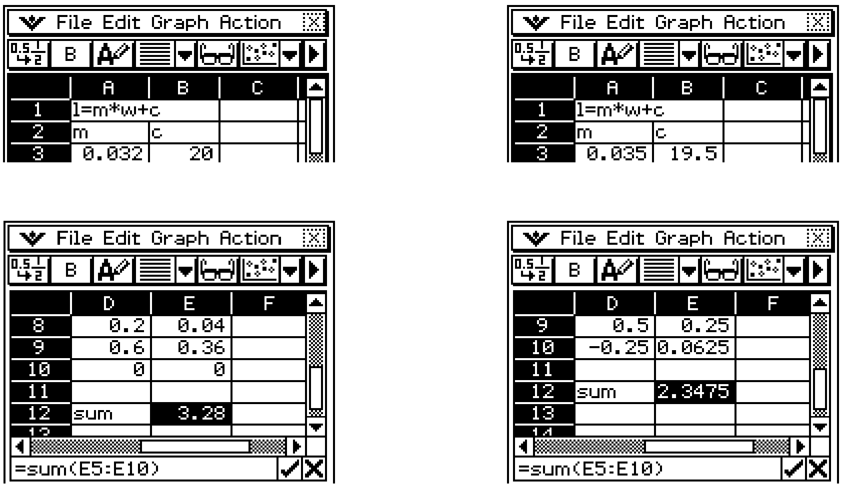


Minimising the sum of squared errors.

One common way of decided the BEST line of best fit is by determining which line has the least sum of squared errors. The spreadsheet can be added to, to make this determination easy.



This addition allows us to confirm that the adjusted values for **m** and **c** (the second model considered on the previous page) provide a better fitting model that the initial values.

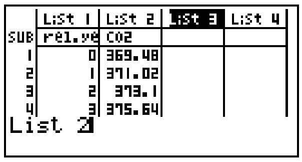


With further refinements to **m** and **c** may provide a better fitting model with a lower sum of squared errors.

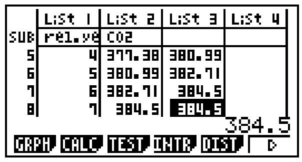
12.3 Working with data and models.

Enter STAT in your CASIO *9860G AU* and enter the data.

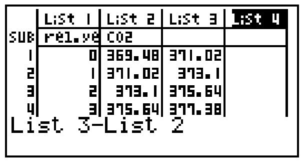
Calculating Additive Change



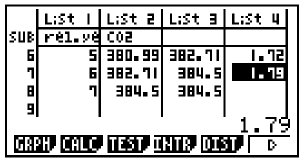
Move the input bar to the header row of `List 3` and enter `List 2` (press SHIFT and 1 for `List`).
This duplicates `List 2` in `List 3`.



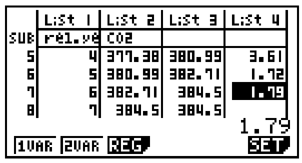
With the input bar on the first element in `List 3`, press DEL.
This aligns the second piece of the data with the first and so on, but creates a space at the bottom of `List 3`.
As we are going to subtract the lists, this space needs to filled with a 'dummy value' like `384.5` (the last value in `List 2`)



The difference between consecutive `CO2` values can now be obtained by calculating `List 3 - List 2`.
This command needs to be entered with the input bar in the header row of List 4.

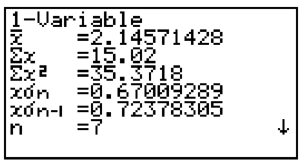


Before these additive change values are worked with further the 'dummy value' (the '`0`' at the List's end) should be deleted.

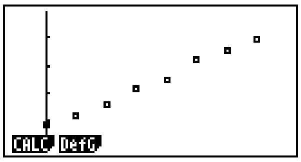


To calculate the average additive change, enter the Calculation sub-menu by pressing `CALC` F1 and then set up the one-variable calculation by pressing `SET` F6.

Ensure that the one-variable `XList` is `List 4` (or wherever you have your additive change values). Once this is done press `1VAR` F1 to see the average/mean $(\bar{x})$ of the additive change values (this is the first of the one-variable calculation outputs).

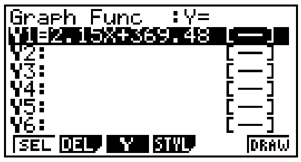


Working with user-defined functional models.

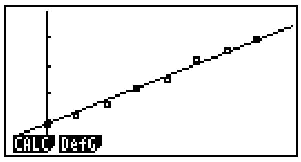


Once we have a possible functional model for data, we can draw that model within STAT to examine its 'fit'.

With a Scatter Plot of the data drawn we have the option to Define Graph by pressing DefG [F2]. This opens up a GRAPH screen where we can enter our linear model.



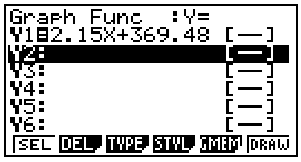
When we press DRAW [F6], the graph of the model is drawn on the Scatter Plot of the data. We can then refine and graph further models on our Scatter Plot.
(if you want to draw a different model on a 'clean' Scatter Plot you will need to [EXIT] and re-draw the Scatter Plot).



Calculating with functional models.

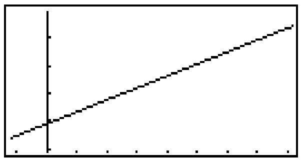
Once a satisfactory functional model has been obtained, it can be used to calculate the value of extrapolations and interpolations. This can be done in many ways, one of which is by using GRAPH.

With a function entered and selected in GRAPH it can be drawn on the existing View-Window by pressing DRAW [F6].

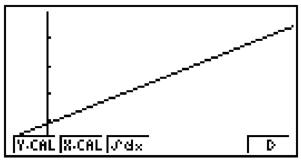


Note 1: a function entered in DefG (or other modes) will not be selected upon entering GRAPH
(press SEL [F1] to Select – check for the black box around the = sign).

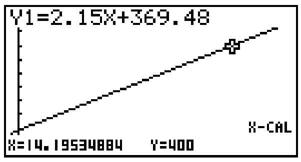
Note 2: The View-Window does not change when moving from STAT to GRAPH. This is quite handy when a Scatter Plot of data is drawn prior to a model for the same data.



To calculate **x** and **y** values for a functional model the 9860's G-Solve menu is very useful.
Press [SHIFT] and then G-SLV [F5] and then ▷ then either Y-Cal [F1] or X-Cal [F2] to calculate a **y** or an **x** value.

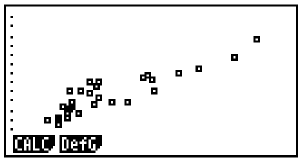


Note: if the desired point is not in your View-Window you will be told it is Not Found, whereupon you will need to broader your View-Window to use this method.

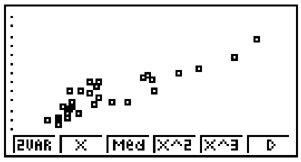


12.4 Obtaining a calculator-defined functional model.

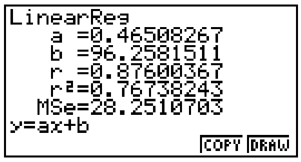
With data sets where it is difficult to determine a value for average additive change, it is sometimes desirable to use a 9860 to obtain a functional model of best fit.



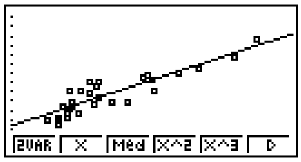
To do this, first draw a Scatter Plot of the data and then press CALC [F1]. This presents a range of the types of functional models that can be obtained by the 9860. To obtain the equation of a linear model of best fit, press X [F2].



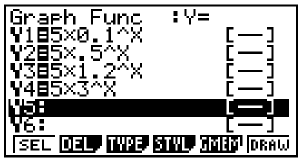
This screen shows you the co-efficients of the linear function of best fit – calculated by the 9860 by minimising the sum of squared errors (the value of the minimum sum of squared errors is also provided as MSe=).



This screen also allows you to COPY [F5] this function into GRAPH and also to DRAW [F6] the graph of this function on the Scatter Plot to assess its suitability.



12.5 Investigating a family of functions.

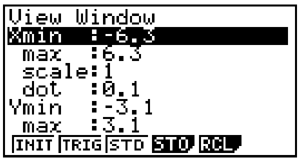


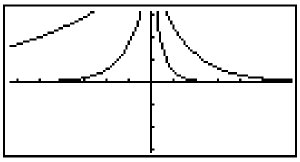
The graphs of a number of members of a family of functions like $y = 5 \times b^x$ can be done in GRAPH.

Enter their equations and press DRAW [F6].

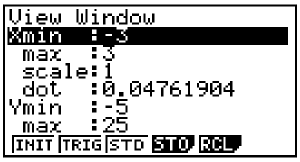
The appearance of such a family of functions depends a lot on your View-Window.

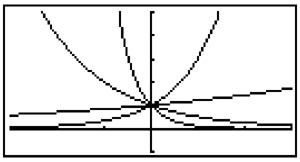
Using the Initial View-Window ([SHIFT] and V-Win [F3] and then INIT [F1]):





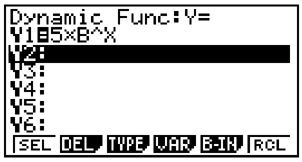
Using a smaller range of x-values and a larger range of y-values, to take into account the rapid growth of this family of function ([SHIFT] and V-Win [F3] and enter values):





This same family can be represented dynamically in DYNA.

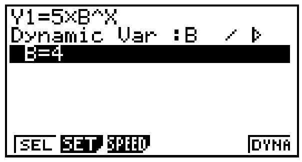
In this mode the function can be entered as shown (press [ALPHA] and then [log] to get the parameter B).



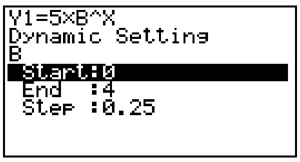
The values for B are set by pressing VAR [F4]

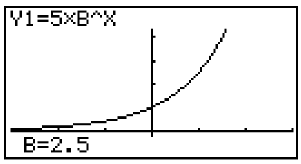
Now press SET [F2] to set the values that B will take

(i.e. the members of the family that will be included)

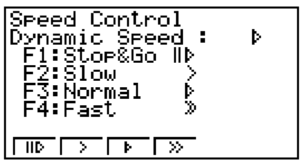


With the settings below you will see, if you wait for One Moment, 17 members of the family displayed dynamically, illustrating the effect of increasing B when graphing members of the family of functions $y = 5 \times b^x$.





Note: Pressing SPEED [F3] allows you to select the speed at which the animation runs from these options:



13. Answers.

4.1 Can you ....1.

1. $A = 180n - 360$

2. a. 180

b.
This value represents the increase in internal angle sum upon the addition of a side to a polygon.

3.
An axis intercept of -360 means that, according to this relationship, a polygon with *no* sides would have an internal angle sum of -360^o (a somewhat "un-real" result).

4.
$A = 180 \times 12 - 360$

$= 1800^o$

5.
$1680 = 180n - 360$

$\Rightarrow n = 11.33...$
But $n \in Z$ (as polygons only have whole numbers of sides).

6.
Rely of the fact that the sum of the interior angles of a triangle is 180^o.

5.1

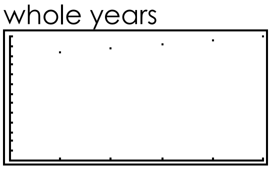
1.
t - the length of the loan in years – is the explanatory variable.
A - the amount owed in $ - is the response variable.

2.

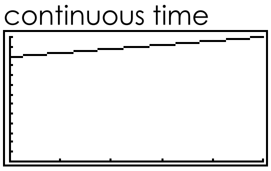
t	A
0	5400
1	5616
2	5832
3	6048
4	6264
5	6480

3.
For each addition of one year there is an addition of a fixed amount ($216) to the amount owed.

4.
If you consider only whole years



If you consider interest accruing over continuous time



5.
$A = 216t + 5400$

6.
a.

$216 \times 10 + 5400$

$= \$7560$

b. $\$7074$

7.
$216t + 5400 = 9000$

$\Rightarrow t = 16\frac{2}{3}$
(i.e. 16 years and 8 months).

5.2

1.
A constant adder in K (i.e. 36) for a constant adder in M (i.e. 10).

2. $K = 3.6M$.

3. a. 66.96 km/h

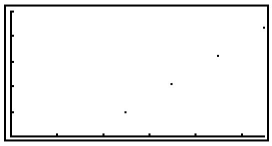
b. 4320 km/h

c. $16\frac{2}{3}$ m/s

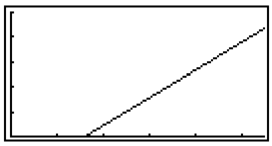
d. $43\frac{1}{18}$ m/s

5.3

1.
This shows the 4 points



This better captures the relationship.



2. $\dfrac{\frac{100}{9}}{20} = \dfrac{5}{9}$.

3. $C = \frac{5}{9}F - \frac{160}{9}$

4. a.

$C = \frac{5}{9} \times 95 - \frac{160}{9}$

$= 35^o$

b. $182\frac{2}{9}^o$

5.
$\frac{5}{9}F = C + \frac{160}{9}$

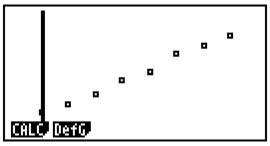
$\therefore F = \frac{9}{5}C + 32$

6. a $32^o F$

b $104^o F$

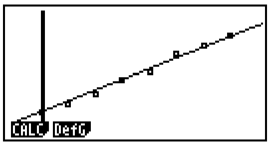
7.1

1.

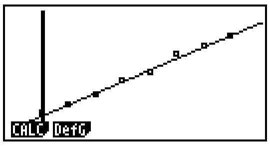


2.
$C = 2.15t + 369.48$

3.



4.
$C = 2.3t + 368.5$



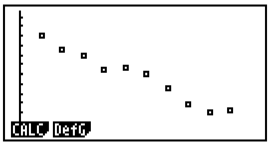
5. 391.5 p.p.m.

6. 2013

7. That the current trend continues.

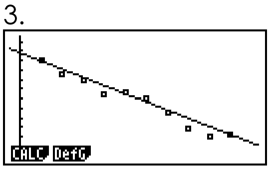
7.2

1.

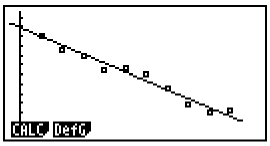


2.
Ave. additive change
$= -42.88.. \div 5$
$= -8.578$
$\therefore y = -8.578x + 1039.889$

3.



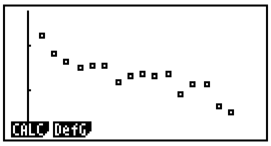
4. $y = -9.3x + 1045$



5. $y = 440.5$

7.3

1.

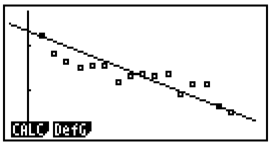


2.
l – an AFL team's ladder position – is the explanatory variable.

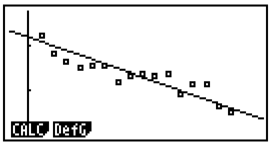
v – the number of Brownlow medal votes earned (in total) by players in an AFL team – is the response variable.

3. $-5.866...$

4.
$v = -5.866l + 117.9$

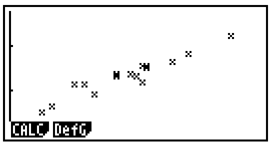


$v = -5.2l + 111$



5.
For every position lower on the ladder a team finished, their player's received approximately 5.2 fewer Brownlow medal votes, on average.

6. (1)

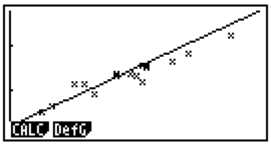


6. (2).
w – the number of wins by an AFL team – is the explanatory variable.

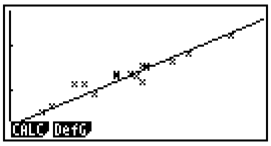
v – the number of Brownlow medal votes earned (in total) by players in an AFL team – is the response variable.

6. (3).
$(5.25 + 5.33 + 2.6 + 17 + 6 - 6 +$
$0.66 + 11.5 + 1 + 12.5 + 7) \div 11$
$= 5.71$

6. (4).
$v = 5.71w + 6.87$



$v = 5.2w + 7$

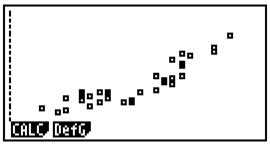


6. (5)
For every additional game an AFL team won, its players received around 5.2 more Brownlow medal votes on average.

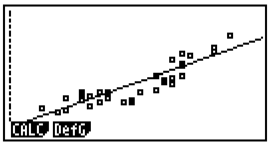
7.
w is a more appropriate explanation of variations in v as;
(1) l is also explained by w.
(2) the graph of w against v shows less non-systematic variation than the graph of w against l.

7.4

1.



2.



3. $p = 5000w - 820$

4.
A diamond weighing 1 carat more will cost \$5000 extra.

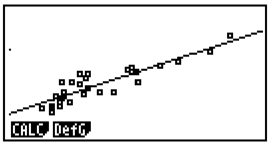
5.
a. $\$1430$
b. $\$1930$
c. 0.564 carats.

7.5

1.
c - the cost of a barrel of crude oil in US dollars is the explanatory variable.

p - the price of a litre of ULP in Australian cents is the response variable.

2.
$p = 0.465c + 96$



3. The "rule of thumb" suggests that the model should have an adder of approx. 1. The model has an adder of around 0.5. As such, a much better rule of thumb would be that a one dollar increase in the barrel price of crude oil adds half a cent per litre to the price of ULP.

8.1

1.
To increase x by 6% is to add 6% of x to x
i.e.
$x + x \times 6 \div 100$
$= x(1 + \frac{6}{100})$
$= x \times 1.06$

2. a. Increase by 45%
b. Increase by 2.5%
c. Decrease by 5%
d. Decrease by 17.5%.

3.
a. $V = 6250(1.085)^n$
b. $V = 14.92(1.0225)^n$
c. $V = 115000(1.11)^n$

10.1

1. $M = 4.2 \times 31.62^R$

2. 7.46×10^{11} MJ

3. 7.46×10^{14} MJ

4.
A 'quake measuring two more on the Richter scale releases 1000 times as much energy!

10.2

1.
A multiplier of 0.85 means depreciation by 15% per year.

2.
$V = 112500 \times 0.85^t$

3. a. $\$22148.37$
b. $\$12540.30$

4. The 20th year.

10.3

1.
$b^4 = 0.89$
$\therefore b = 0.971$
This represents decay of 2.9% per day.

2. $W = 100 \times 0.971^t$

3. a. 81.38 grams
b. 0.0022 grams

4.
$0.971^t = 0.5$.
$\Rightarrow t = 23.6$

10.4

1. 1.259

2. $I = 10^{-12} \times 1.259^d$

3. a. 4×10^{-7}
b. 0.0318

10.5

1.
1000×1.015^{200}
$= \$19643.03$

2. $V = 1000 \times 1.005^t$

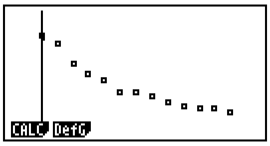
3. $\$19935.90$

4.
$V = 1000 \times 1.000164^t$
$\$20080.59$

5.
Value compounded
quarterly = $\$2.44$
monthly = $\$2.61$
daily = $\$2.714567$
second = $\$2.718281$
Compounded continuously the value $= e = 2.718281828459...$

11.1

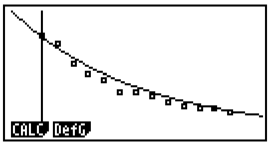
1.



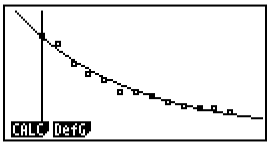
2.
The flow decreases rapidly at first, then more slowly.
Given the context the flow should decay to zero.

3. 0.883

4.
$F = 51.7 \times 0.883^t$



$F = 51.7 \times 0.87^t$

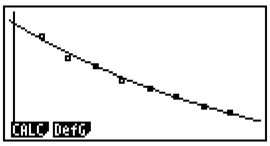


5.
$t = 16.77$
i.e about three quarters the way through October 1999.

7.
As the flow for October was below 5 MMscf/d the well was needed in that month.

8.
The increase in flow suggests the well was ready in November.

9.
$F = 31.5 \times 0.905^t$



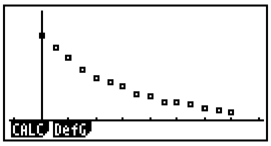
When $t = 19$, $f < 5$
i.e November 2001.

11.2

1.
Adders in $n = 1$
Multipliers in $H = \ldots$

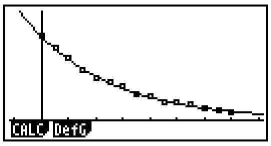
0.86	0.8888	0.9
0.8604	0.8	0.8333
0.8108	0.875	0.8
0.85	0.7857	0.8333
0.8823	0.909	

2.



3.
$H = 10 \times 0.849^n$

4.



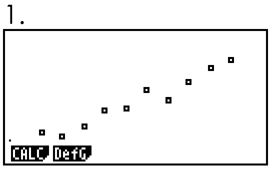
5.
As n, the bounce number, must be a positive integer, no additional heights can be interpolated between bounce 0 and bounce 14

6. 0.37 metres

7. The 29th bounce.

11.3

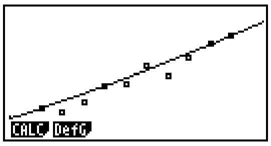
1.



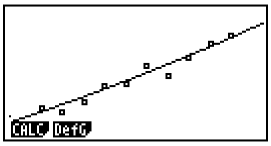
2. 1.026

3.
$103.2 \div 1.026^{10}$
$= 79.8$
Hence
$y = 79.8 \times 1.026^x$

4.



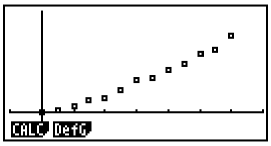
5.
$y = 78 \times 1.027^x$



6. 151.8

11.4

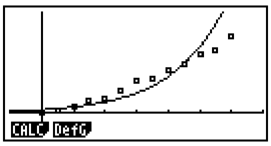
1.



2.
Because there is no constant adder in internet use .

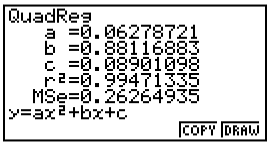
Because the scatter plot is curved!

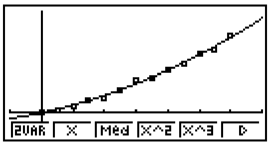
3.
The “best” simple exponential model that can be obtained is something like
$W = 0.93 \times 1.34^t$
But this model fits the data very poorly



The reason that there is no better model of this type is that the data does not have an approximate constant multiplier.

4.





5. In 2012.

6. There is a good chance that the prediction will be inaccurate as there may not be another 13% of the world's population ready to take up internet use (i.e. the trend of increasing growth may be unable to continue).