Growth and Decay in Finance

Preamble

Growth and Decay in Finance has always been possible under the existing curriculum, but will become mandatory from 2016. Therefore, it seems prudent to explore ways of thinking about recursion in finance situations, to enable your students to become comfortable with this fundamental part of finance. The intention here is to be quite informal – a way of thinking about growth and decay in a dollar setting is the goal.

Progressions

A very nice place to start is to consider two simplistic situations.

Firstly, consider a starting value Q, to which a fixed amount k is added or subtracted.

$$Q + k \quad \begin{matrix} \boxed{k > 0} \\ \boxed{k < 0} \end{matrix}$$

Imagine the behaviour of this situation if the addition of k were to be repeated.

Secondly, consider a starting value Q, which is multiplied by a fixed <u>positive</u> value r.

$$Q \times r \quad \begin{matrix} \boxed{r > 1} \\ \boxed{r < 1} \end{matrix}$$

Imagine the behaviour of this situation if the multiplication by k were to be repeated.

The core of where this needs to get is to ultimately imagine these two operations being applied in tandem:

$$Q \times r + k$$

or perhaps

$$(Q + k) \times r$$

depending on which order you require.

In each of these latter situations, the different types of k and r values might be needed.

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Our first finance scenario

Suppose you had a situation where $1000 was placed in a safe location. Then, every year for the next 21 years, a further $250 was added in.

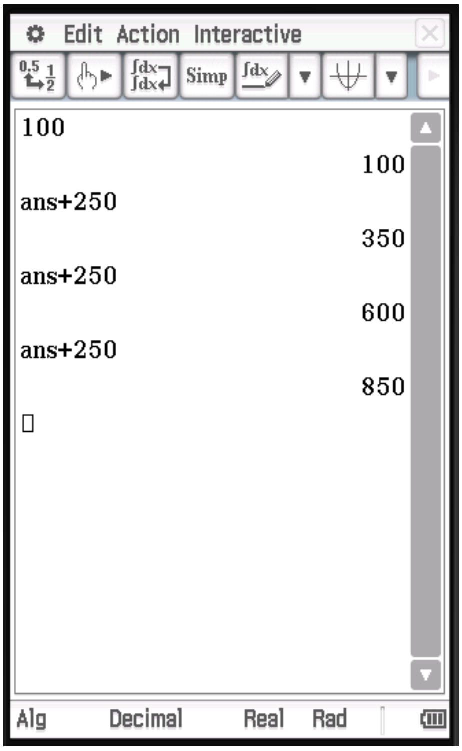
Can you imagine what the situation might be?

Make sure you give this some creative thought…

The idea I had in mind was the birth of a grandson, where the grandparent gets the ball rolling with an initial $1000 deposit, and then every year up until the 21st birthday adds a further $250.

Spend some time now thinking about how this might be modelled using technology, and don't jump too far ahead:

On your ClassPad –



Clearly, there are Pros & Cons to this approach.

The greatest Pro is the very evident structure of the recursion being in evidence.

Do not overlook the power of this, as it is this mental picture we need to cultivate.

The greatest Con is the need to keep track of how many times you have pushed $\boxed{\text{EXE}}$! One interruption at the wrong moment and you start again.

Clearly this can be achieved in other ways, but beware of losing sight of the structure for students – the one-hit formula which gives the answer does not necessarily reveal the reality of the mechanics of the recursion.

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Our second finance scenario

You give Mum $1000.

She will increase this amount by %2 each month (very generously!) in addition to which you give her a further $80 per month of 'Macca's money' (earned flipping burgers!).

Can you imagine how this amount grows? Can you predict the amount at a future time?

Have a go at modelling this.

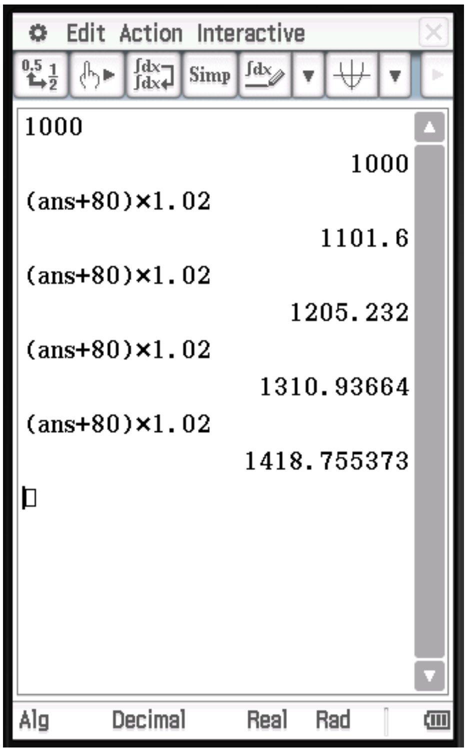
Now, interestingly, there are two approaches here, which depend very much on the order of events, and the rules of the game.

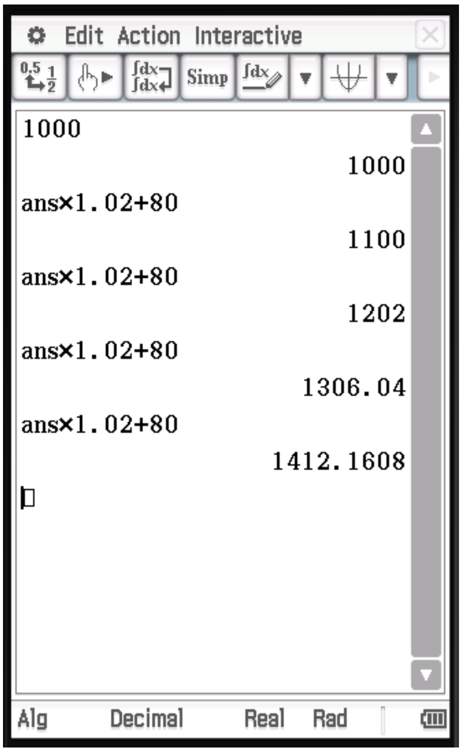
Either we get

$$(1000 + 80) \times 1.02$$

Or

$$(1000 \times 1.02) + 80$$





It is very important that you use this opportunity to explore a few things. Firstly, make sure that the 1.02 multiplier is understood to be the equivalent of increasing an amount by 2%. Secondly, in a realistic context, what do the banks do? Unless your investment is in an account where interest is accrued daily, the chances are that a commercial bank will calculate interest on the minimum monthly balance – so the interest will be calculated before the extra deposit is taken into account. Finally, consider Mum's generosity – what is the annual equivalent of 2% interest every month?

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Our third finance scenario

You buy yourself a car.

In order to do so, you need to borrow $25000 from a bank.

The bank charges you 6% interest per annum, compounded monthly. That means, the interest will be calculated each month based upon what you owe at that time. If the annual interest rate is to be calculated 12 times per year, then the accepted practice by banks is to divide that interest rate into 12 equal-sized pieces, so

$$6\%\ per\ annum \text{ equates to } \frac{6}{12}\%\ per\ month\text{, being } 0.005 \text{ as a decimal.}$$

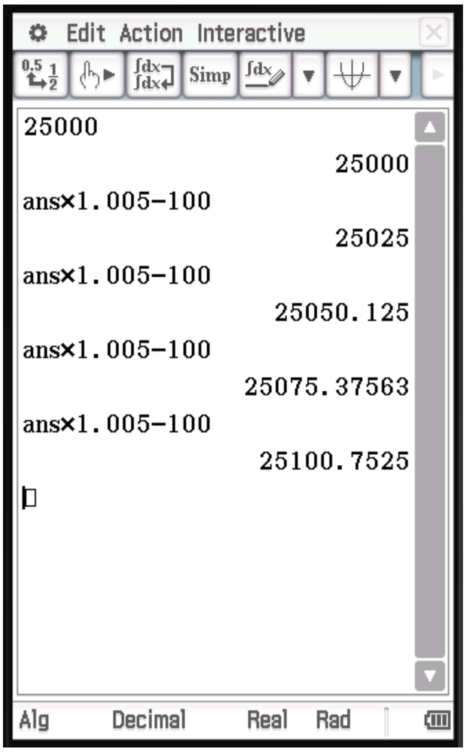
You can pay back $100 per month.

Once again, commercial banks are in the business of making money – so this time, they charge interest on the maximum amount owing in the month. So, our calculation looks like

$$(25000 \times 1.005) - 100$$

Make sure you understand why the multiplier comes first, and why a bank might choose to do so.

On your Classpad:



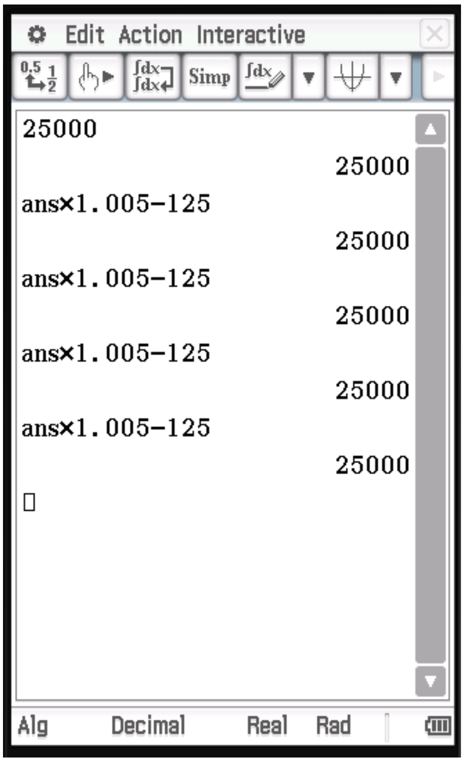
What the heck? The outstanding balance is going up! You are never going to pay back the amount – every month, the amount you owe is increasing. The bank will steadily make money out of you, and you will end up owing much more than you originally borrowed, which does not seem fair.

Perpetuity

Let's increase the regular payment to $125, to see if that changes things:

$$(25000 \times 1.005) - 125$$

On the ClassPad:



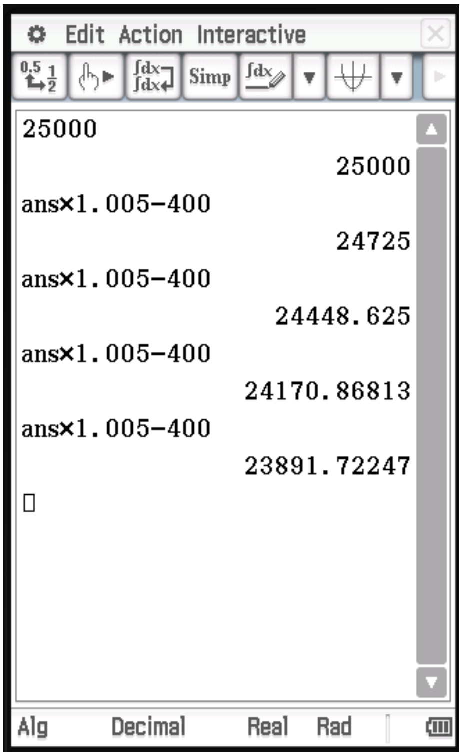
Here we see the phenomenon of Perpetuity in a straightforward sense. A break point or tipping point has been reached.

The amount of interest being accrued is equal to the payment – effectively the amount owing stays the same.

We do not see this much in load scenarios in Australia, but it is very common overseas. For example, in the UK, it is possible to do exactly this for a home loan – payments to exactly equal the interest. In those cases, banks typically demand a separate investment (equal and opposite) which will mature with the value of the loan at a specified time in the future, at which point the value of the investment is used to completely pay off the debt.

Paying off the Loan – The Reducing Balance Loan

Clearly, then, our repayment must exceed the regular interest which is charged. If that is the case, we will see a Reducing Balance Loan:



Notice that the outstanding balance is not reducing by a regular amount.

Invite students to mess about to genuinely comprehend the mechanics of these situations – they will typically be surprised at how slowly the outstanding balance reduces, especially if you make the Principal (the initial amount borrowed) as large as a typical home loan.

Even worse, invite them to consider the total amount paid back over the 30 years of payments!

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A Formal Treatment

To see the algebraic treatment of our scenario above in an iterative sense, we have

$$Q_0 = 25000$$

$$Q_1 = 25000 \times 1.005 - 400$$

$$Q_2 = Q_1 \times 1.005 - 400$$

$$Q_3 = Q_2 \times 1.005 - 400$$

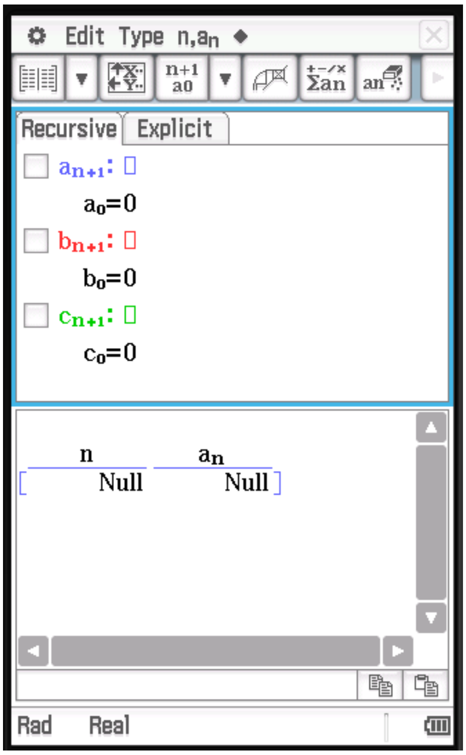
....

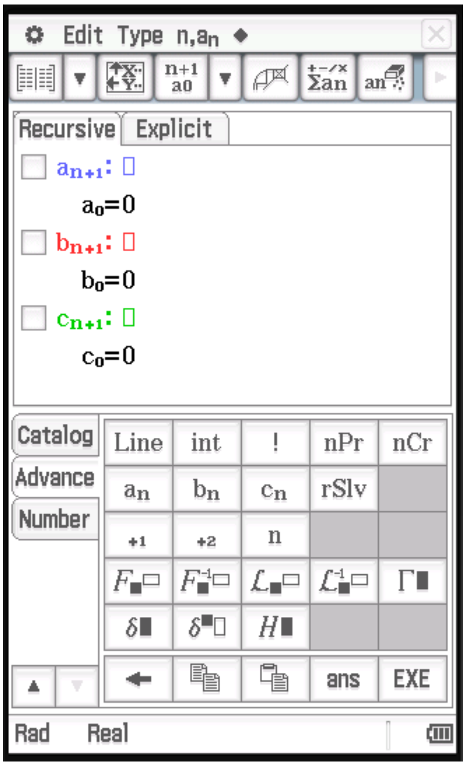
$$Q_{n+1} = Q_n \times 1.005 - 400$$

For those who would like to see this formalised version on the Sequence (Recursion) application of the ClassPad, here is a summary:

Choose Sequence from the Main Menu, and then if your top entry does not say a_{n+1} and a_0 then use the downwards pointing triangle alongside the $\frac{n+1}{a0}$ on the menu bar option, indicating a list box, and choose the top option, using n+1 and a_0. The effect of this is that when we generate our sequence of values, it will start with a first term of a_0 which we will think of as the initial value of the investment.

The n+1 part indicates that our next term of the sequence, a_{n+1} , can be found using operations acting on the last known term, a_n .





In order to define the iteration required to find the next term, we define how a_{n+1} can be found from a_n .

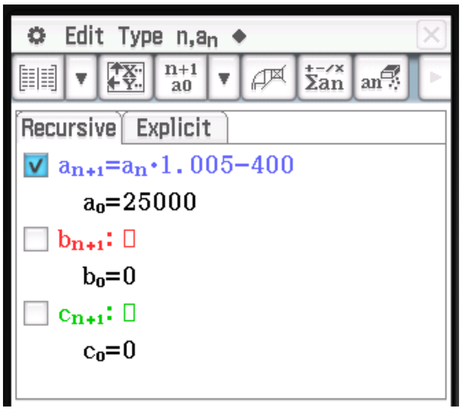
Your cursor should be flashing alongside the a_{n+1} :

We need to input our earlier iterative relationship, being that

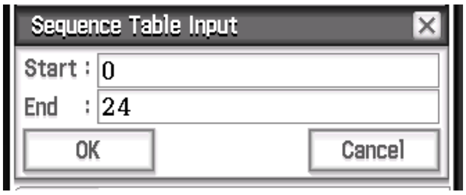
$$a_{n+1} = a_n \times 1.005 - 400$$

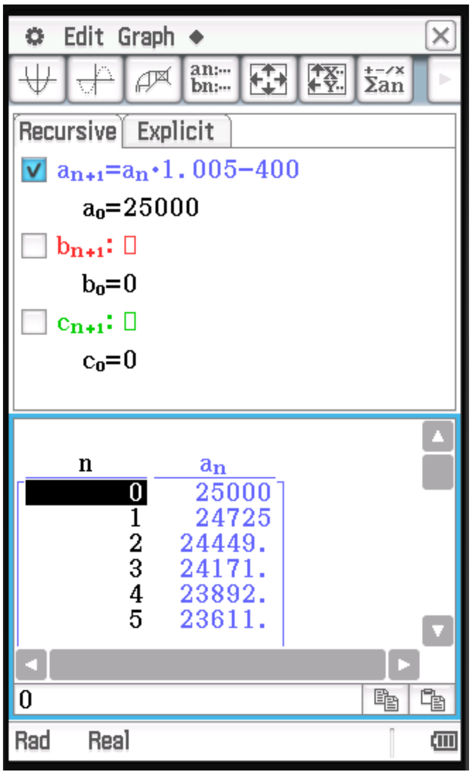
Either use the Advance soft keyboard or the n,a_n menu tab to enter the required a_n in the definition.

Also enter the required value of a_0



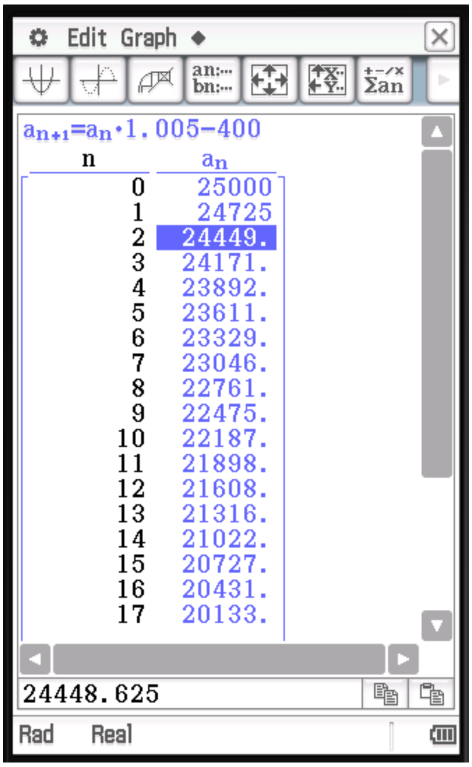
Now, specify the number of sequence member terms you would like to see in a table, by pressing the icon, which presents the input form shown at right:





Pressing OK presents you with the screen at left, showing the early sequence members.

Resize the lower half of the screen using the Resize option from below the screen, and the first 18 sequence members will be presented in scrollable format.

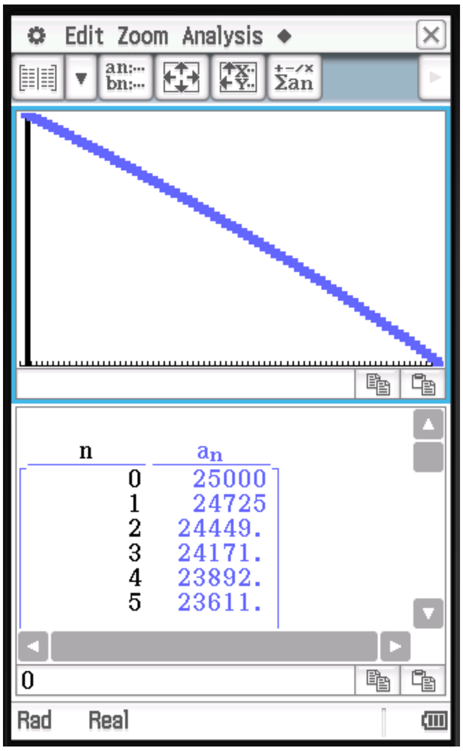


Do not fear the use of this iterative, or recursive, area of your ClassPad. It illustrates beautifully the mechanics of the situation, and can be used for a variety of purposes.

Up at the top of the screen, the recursive relationship is presented.

Note that in finance it is common practice to start with term zero (Q_0 or a_0). This allows the term numbers of subsequent terms to match the number of time periods after the beginning of the loan payment program, so a_{12} at left (21608.72) is the amount owing after 12 time periods.

We can visualise this by plotting points – the plot shown to the right is the result of plotting the first 75 members of the sequence, which will pay off the loan. Notice the concavity. This is not a straight line.



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Superannuation and Retirement – the Reducing Balance Investment

Consider the case of a typical worker. Over the course of their working lives, they pay into a superannuation fund, which operates just like the earlier situation where regular deposits are made as well as regular interest calculations providing a little extra.

If such a worker retired with a super fund worth $1 000 000 (that's one million dollars!) which is in an investment account which pays 3% interest per annum then how long will it last if they would like to spend $10 000 every month? Assume interest is calculated monthly, so the monthly interest rate will be

$$3\%\ per\ annum \text{ equates to } \frac{3}{12}\%\ per\ month\text{, being } 0.0025 \text{ as a decimal.}$$

Our iterative process here looks very much like a reducing balance loan, only this time the balance is your own money that the bank is looking after, rather than the amount of money you owe the bank. You can think of it as the reverse process, only this time the regular amounts come from the bank to you rather than the other way around. It does of course mean that they will want to calculate the interest in such a way as to minimise what they need to give you, so this time the minimum monthly balance will be <u>after</u> you pay yourself your monthly spending money. The order of operations needs to be, "take out some money first, then calculate the interest paid on the remaining balance"

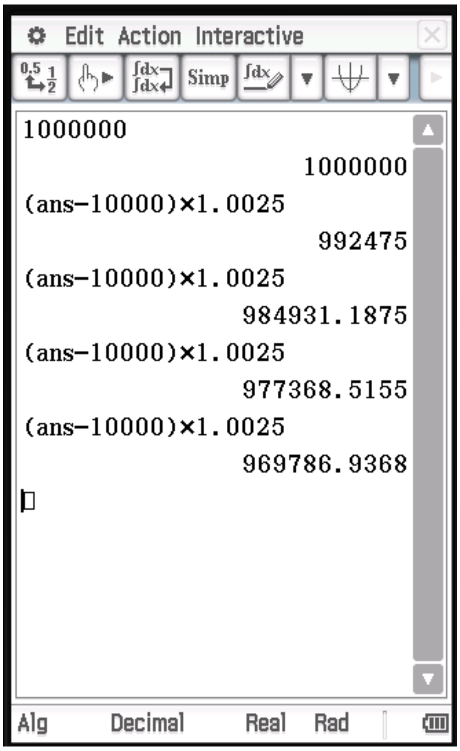
So, we have an initial withdrawal looking like this:

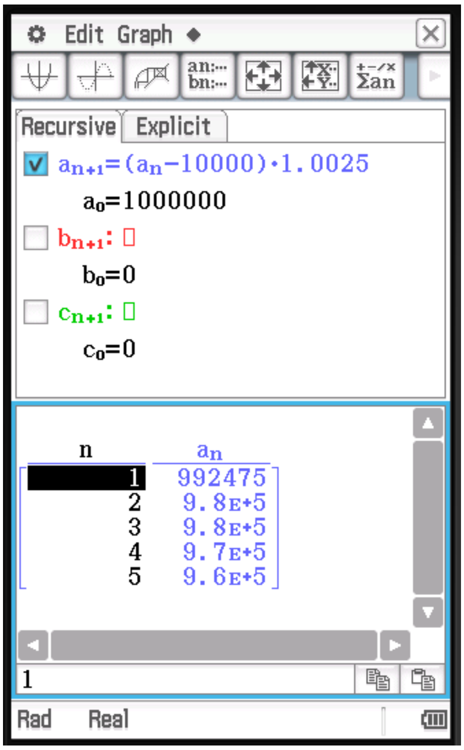
$$A_0 = 1000000$$

$$A_1 = (A_0 - 10000) * 1.0025$$

$$A_2 = (A_1 - 10000) * 1.0025$$

With the ClassPad, in main to the left and in Sequences to the right:





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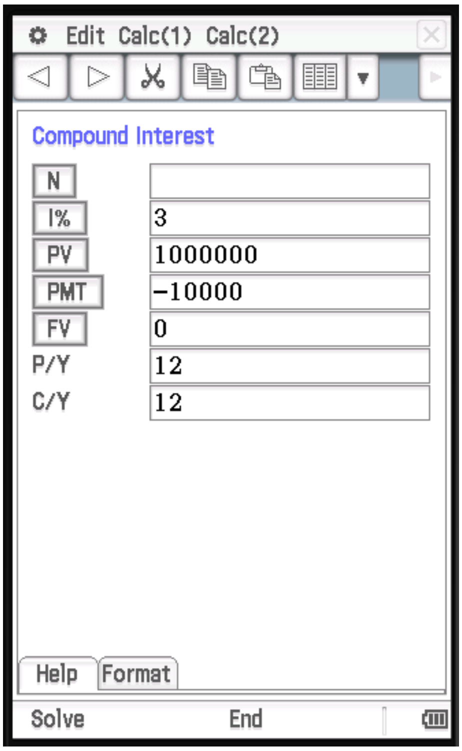
At this point, you have yourself quite a laborious task to find how long the money lasts – unless you are prepared to embrace Annuities and their formulas.

I will do so on the following pages, but at this point it is worth mentioning the powers and the dangers of the Financial App which is always available.

Firstly, the Financial App operates following "cash flow" logic – think credits and debits – and you must take either your perspective or the bank's perspective. This effectively means you must see money flowing in one direction as a positive, and in the opposite direction as a negative. The bank sees money coming in as a positive, and money going out as a negative. You see it the other way around.

The Financial App is extraordinarily powerful, but is only for confident and strong kids. I would not recommend it to be used for any simple interest calculations. If you use it for Compound Interest, it can be very good at solving any scenario that you give it – however you must be clear on what you are asking it to do.



A field at a time:

N is the number of interest calculation periods

I% is the annual interest rate (regardless of how many times it is calculated per year)

PV is the present value, or Principal, or starting balance

PMT is the size of any regular payment

FV is the Future Value, or final balance

P/Y is how many payments there are per year (P for payments)

C/Y is how many recurring interest calculations there are per year (C for compounding)

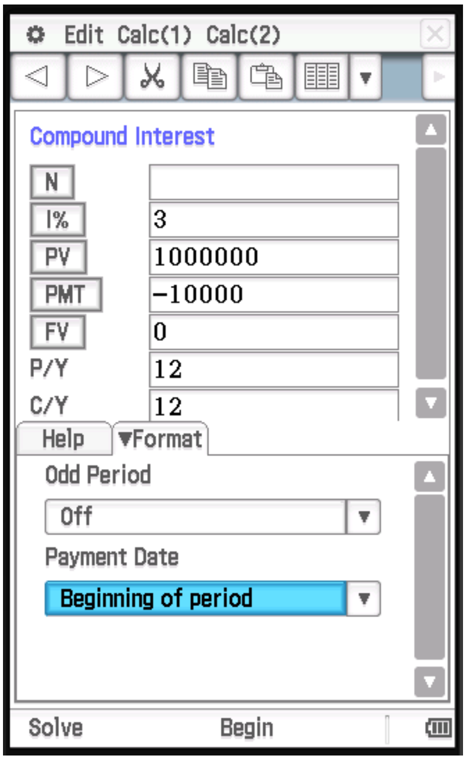
The idea here is that you fill in all fields that you know, but one. Click on the button alongside the one you would like to calculate, and it will solve it for you.

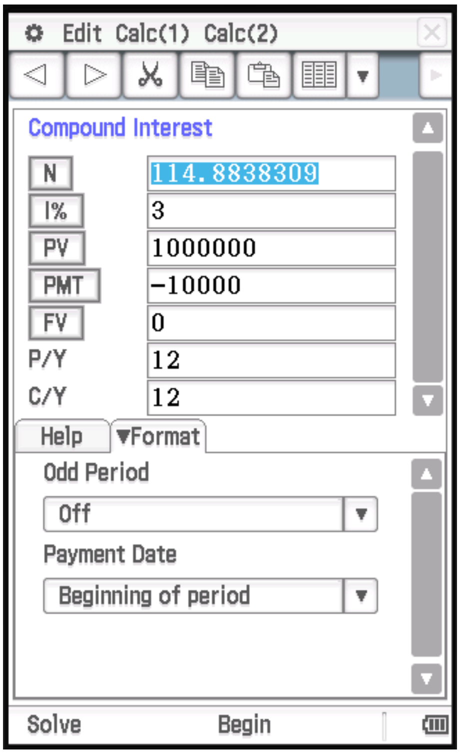
The input form above has been filled in with all the relevant fields for our retirement scenario. The N value is left blank, as it is the value we are after.

Before we proceed – this area of the calculator by default is used for compound interest seen as growing investments – the superannuation fund itself, where the balance grows. In those cases, as mentioned earlier, the bank wants to pay interest on the minimum balance in the account per interest period, which will be the amount before the next payment is included. The format tab on the lower part of the screen will therefore say 'Payment Date – End of period' by default. If we leave it this way, our calculation will actually not match with a real bank treatment, as they want to calculate the interest after you take your money out, so they pay out less interest.

Effectively, the assumption is that you take out your first lot of money in the first month that the retirement fund is set up, so it needs the first calculation to be (1000000 – 10000)*1.0025 indicating the money is paid out before the first interest accrual. We need to change this to 'Payment Date – Beginning of period" so that the calculation reflects this desire to remove the first payment prior to any interest calculation.

Then, we press the N button, and are presented with the following result:





Our retirement fund will last for 114 months. That is 9 and a half years.

As the actual value for N calculated is 114.88, what in reality will happen is that the bank will do the calculation for 114 months, then your last payment will be whatever remains in the fund, after which the balance will be zero. This last payment will be less than $10 000.

You could, if you wanted to, use this to calculate the amount you could take out each month if you wanted the fund to last for 20 years.

Looking to the right, you will see that you can afford to take out a little over $5 532 per month to make the fund last 20 years.