
**Do you know all there is
to know about using your
graphics calculator in the
SACE Mathematical
Methods Examination?**

Specifically the CASIO fx-9860G AU PLUS
2nd Edition

Written by Elena Zema
Edited by Anthony Harradine
and Alastair Lupton

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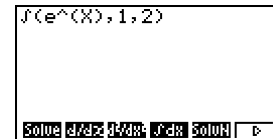
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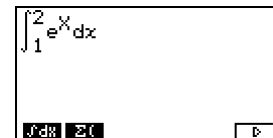
LINEAR MODE VS MATH MODE

The 9860 has two operating modes: Linear Mode and Math Mode.

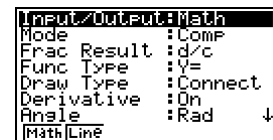
Linear Mode is the traditional mode of scientific calculators where calculations are entered in a line, as seen opposite.



Math Mode allows the user to work with calculation entry that looks 'natural'. This is often termed natural input. It can be seen opposite.



To set your 9860 to work in the mode you desire, enter the SET UP when in the RUN•MAT (RUN•MAT) application. To do this press **SHIFT** then **MENU**. You can then choose the option you want.



Math Mode is the default setting for 9860's using OS version 2.0 and later.

In this book all instructions assume you have your 9860 set to work in Math Mode.

WORKING WITH STATISTICS

NORMAL DISTRIBUTION

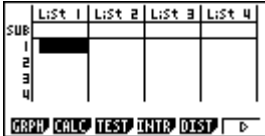
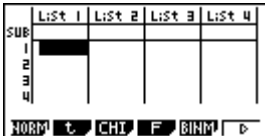
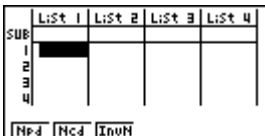
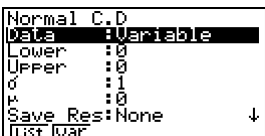
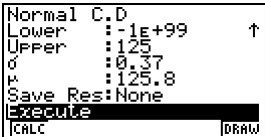
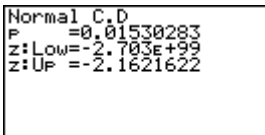
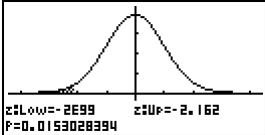
How do I find probabilities from normal distributions?

Example:

A certain brand and flavour of chip is sold in bags stamped as containing 125 grams. The chips are delivered to each bag by a machine and the weight varies from packet to packet. Let W denote the weight of the chips in a packet. The distribution W can be modeled by a normal distribution with a mean of 125.8 grams and a standard deviation of 0.37 grams.

Determine the proportion of packets that will contain less than 125 grams of chips.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  . Press F5 to access the DISTribution menu.		
Press F1 to access the NORMal distribution menu.		
Press F2 to access NCD (normal cumulative distribution) function.		
Input the values. Press EXE after each entry.	 	In this example, we want to find $P(W < 125)$. The lower limit of $-1E99$ is used to reflect the characteristics of the normal distribution. To find $P(120 < W < 125)$ set Lower to 120 and Upper to 125.
Once complete, press F1 (or EXE) to CALCulate the probability. The probability will be displayed along with the corresponding Z scores. Alternatively, press F6 to DRAW the normal distribution and shade the region of interest, as well as display the probability along with the corresponding Z scores.	 	Therefore, $P(W < 125) = 0.0153$ (or 1.53%) - correct to 3 significant figures.

 Note that, when using some older operating systems, the use of $-1 E 99$ or $1 E 99$ as the lower or upper limit may at times result in the calculator returning Ma Error. Should this occur, use $-1 E 90$ or $1 E 90$ (or a similar suitably small or large number).

INVERSE NORMAL

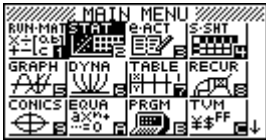
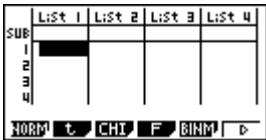
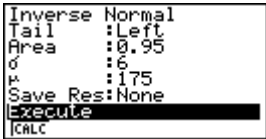
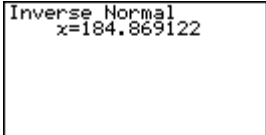
How do I find a score, given a normal probability value?

Example:

The height of all year 12 female students is assumed to be normally distributed with a mean of 175 cm and a standard deviation of 6 cm.

Find how tall Elena is if she is taller than 95% of year 12 students?

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  .		
Press F5 to access the DISTRibution menu.		
Press F1 to access the NORMAL distribution menu.		
Press F3 to access InvN (inverse normal) function.		
Input the values. Press EXE after each entry. Once complete, press F1 (or EXE) to calculate the value of the score.		In this example, we want to find x such that $P(X < x) = 0.95$. So the area we know is to the left of the unknown score. Hence we set the "Tail" setting to Left. Had we wanted to compute $P(X > x) = 0.2$, we would enter 0.2 for the area and set the "Tail" setting to Right.
The value of the score will be displayed.		Therefore, $x = 185$ cm correct to 3 significant figures.

CONFIDENCE INTERVALS FOR THE MEAN

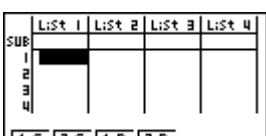
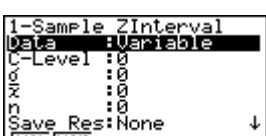
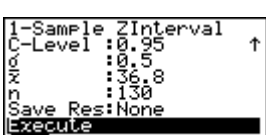
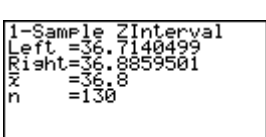
How do I find a confidence interval for the population mean?

Example:

In 1990 a study was undertaken in which the body temperature of 130 healthy men from one nation were measured. The mean temperature of the sample was found to be 36.8°C . Let μ be the mean body temperature of all healthy men from this particular nation in 1990.

Assuming the population standard deviation is 0.5°C , find a 95% confidence interval for μ .

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application 		
Press F4 to access the INTRerval menu.		
Press F1 to access the Z menu.		
Press F1 to access the 1-S (one sample) function.		
Press F2 to select VARIABLE.		
Input the values. Press EXE after each entry. Once complete, press F1 (or EXE) to calculate the interval.		Note that the standard deviation of the population is entered, not the standard error.
The confidence interval will be displayed.		

BINOMIAL DISTRIBUTION

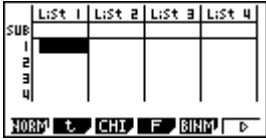
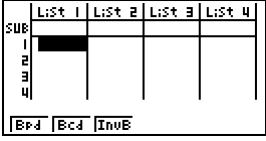
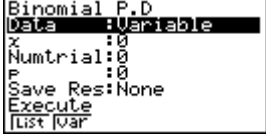
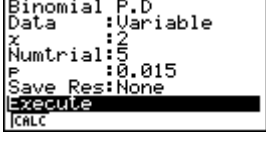
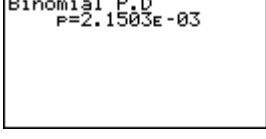
How do I calculate probabilities (non-cumulative) from a binomial distribution?

Example:

The label on each packet of a certain brand and flavour of chip states each bag contains 125 grams of chips. Suppose that 1.5% of all bags produced contain less than 125 grams of chips. Let Y be the number of packets in the random sample that contain less than 125 grams of chips.

A random sample of 5 packets is selected. Use the binomial distribution to find the probability that 2 packets will contain less than 125 grams of chips?

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application () and then press F5 to access the distribution menu.		
Press F5 to access the BINomial distribution.		
Press F1 to access Bpd (binomial point distribution) function.		
Press F2 to select VARIABLE.		
Input the values. Press EXE after each entry. Once complete, press F1 (or EXE) to calculate the probability.		In this example, we want to find $P(Y=2)$, where Y can be modeled by $B(5, 0.015)$.
The probability will be displayed.		Therefore, $P(Y=2) = 0.00215$ or 0.215% (3 significant figures).

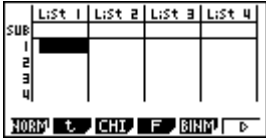
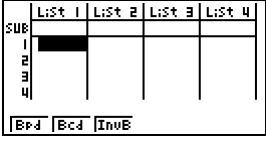
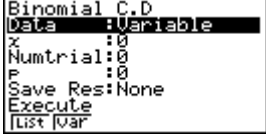
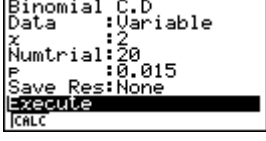
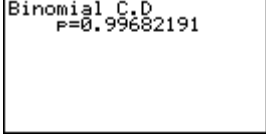
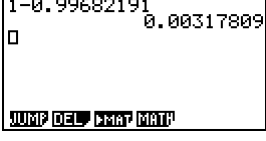
How do I calculate probabilities (cumulative) from a binomial distribution?

Example:

The label on each packet of a certain brand and flavour of chip states each bag contains 125 grams of chips. Suppose that 1.5% of all bags produced contain less than 125 grams of chips. Let Y be the number of packets in the random sample that contain less than 125 grams of chips.

A random sample of 20 packets is selected. Use the binomial distribution to find the probability that **more than 2 packets** will contain less than 125 grams of chips?

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application () and then press F5 to access the distribution menu.		
Press F5 to access the BINomial distribution.		
Press F2 to access Bcd (binomial cumulative distribution) function.		
Press F2 to select VARIABLE.		
Input the values. Press EXE after each entry. Once complete, press F1 (or EXE) to calculate the probability.	 	The Bcd function only calculates "less than or equal to" probabilities. In this example, we want to find $P(Y > 2)$, where Y can be modeled by $B(5, 0.015)$. So first we must compute $P(Y \leq 2)$ and then subtract from 1.0
We can now compute $1 - P(Y \leq 2)$.		

CONFIDENCE INTERVALS FOR THE PROPORTION

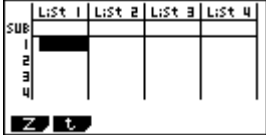
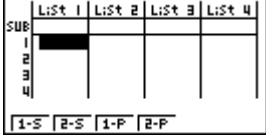
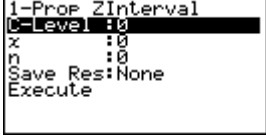
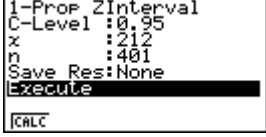
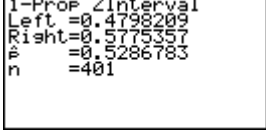
How do I find a confidence interval for the population proportion?

Example:

The Gawler council decides to close Junction Road. A random sample of 401 residents from Gawler was asked whether they thought the road should stay open. 212 of the 401 surveyed said the road should stay open to traffic.

Calculate an approximate 95% confidence interval for the proportion of all Gawler residents who thought the road should stay open.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application () then press F4 to access the INTRval option.		
Press F1 to access the Z menu.		
Press F3 to access the 1-P (one proportion) function.		
Input the values. Press EXE after each entry.		
Once complete, press F1 (or EXE) to calculate the interval.		Note that x is the number of "successes" in the sample (supporters in this case).
The confidence interval will be displayed.		

ALGEBRAIC MODELS FOM DATA – WORKING FROM OBSERVATION

ALGEBRAIC GENERATION OF LINEAR MODELS

How do I obtain, assess and use a linear model?

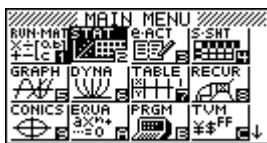
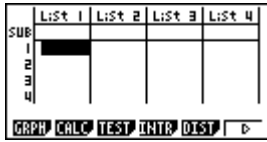
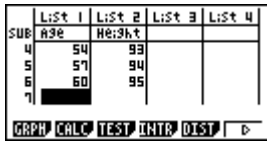
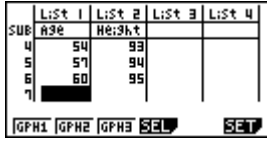
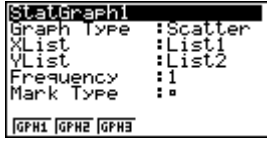
Example:

Basil's height has been recorded at particular points in time. The data is presented in the table below:

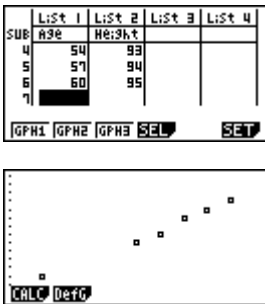
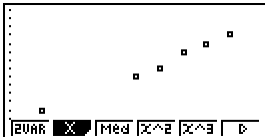
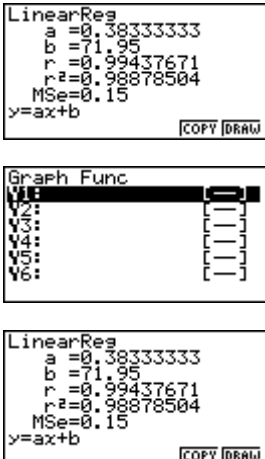
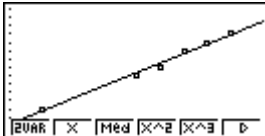
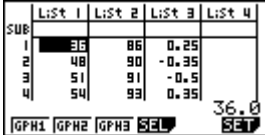
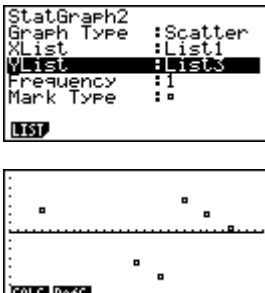
Age (months)	36	48	51	54	57	60
Height (centimetres)	86	90	91	93	94	95

- Make a scatterplot of these data.
- Determine the equation of the linear model that best fits these data.
- Make a residual plot to help assess the appropriateness of the model.
- Predict Basil's height at the age of 65 months.
- According to the model, how old will Basil be when he is 89 cm tall?

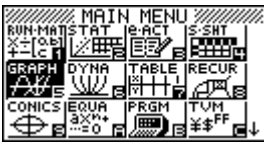
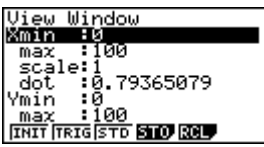
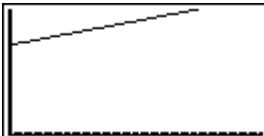
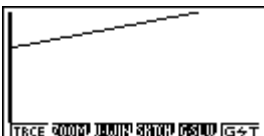
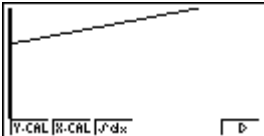
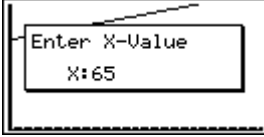
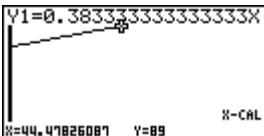
PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  .		
Enter the SET UP screen of the STAT application by pressing [SHIFT] then [MENU] . Scroll down to the Resid List setting and press LIST (F2) and then enter [3] to set the residual list to be List3. When computed, the residuals will be placed in List3.	 	Set the Stat Wind option to Auto. This will see the 9860 automatically choose the best view window setting for the data entered. Also note that the List File is set to File1. The 9860 has six files (a file is a set of twenty six lists) to choose from.
Enter the age data into List 1 and the height data into List 2. Press [EXE] after each entry.	 	You can name the lists to suit the data. Type in the variable name into the SUB line. When entering data, it is a good idea to place the independent variable's data (x) into List 1 and the dependent variable's data (y) into List 2.
Select GRPH (press [F1]). Press [F6] to select SET. Scroll down to adjust the StatGraph1 options. For a scatterplot, scroll down to Graph Type and press [F1] for Scat. Check the correct lists are set for the x and y axes. Press [EXE] or [EXIT] to return to the stat list window.	 	The process is continued on the next page.

PROCESS (cont.)

ACTION	DIAGRAM	COMMENTS
From the stat list window, use GPH1 (F1) to make the scatterplot. The scatterplot is displayed.		You can also use the arrow keys, (◀, ▶, ▲, ▼) to move the view window.)
To determine the equation of the linear model of best fit press CALC (F1), then select the linear option, X (F2), then choose the form for your linear model, $ax+b$ (F1) or $a+bx$ (F2).		In the work below, the form $ax+b$ (F1) has been selected.
The output screen displays the slope and intercept of the equation of the linear model of best fit, as well as Pearson's product-moment correlation coefficient, r, and the coefficient of determination, r^2. The COPY option (F5) allows you to copy the equation of the linear model to both the Graph and Table applications. This is very useful for further analysis. In this case we have selected Y1 and then pressed EXE.		Note that you can choose to copy the model into any of the Y positions, particularly if you have other functions defined.
Press F6 to DRAW the graph of the linear model on the scatterplot.		
Now press EXIT twice to return to stat list window and you will see that List3 has been populated with the residual values.		
You can now SET (F6) StatGraph2 to be a scatterplot of List3 vs List1 to make a residual plot.		This process is continued on the next page.

PROCESS (cont.)

ACTION	DIAGRAM	COMMENTS
To predict values using the linear model: Enter the GRAPH application .		You could also use the Table application () for this step.
The linear model of best fit was copied into Y1 earlier. Note that it will be unselected and so you will have to press SEL (F1) to select it for graphing.		
Before drawing the equation, you should always check your view window settings. Press  then  to check/change your settings. The setting will currently be those used for the residual plot. Change them to those shown opposite. Press  and then DRAW () the function.	 	
To find a predicted y-value for a given x-value do the following. Press G-SOLVE (, ) to view options. Then press () view the rest of the menu. Use Y-CAL () . Enter the x-value and press . Both the x and y values will be displayed.	   	You could also find the predicted value in the RUN application. Press . Select GRPH () . Select Y () then enter the x-value. Close bracket and press . 
To predict an x-value for a given y-value press G-SOLVE (, ) to view options. Then press () view the rest of the menu, use X-CAL () . Enter the y-value and press .		

ALGEBRAIC GENERATION OF EXPONENTIAL MODELS

How do I obtain, assess and use an exponential model?

Example:

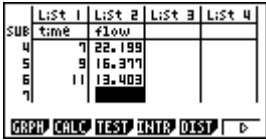
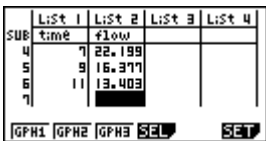
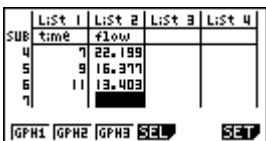
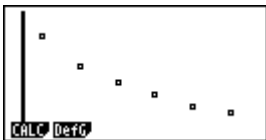
The table below gives the average daily flow rate from a gas site for the months shown.

Relative time, t (months)	1	3	5	7	9	11
Rate of Gas Flow, f (MMscf/d)	51.717	36.717	28.066	22.199	16.377	13.403

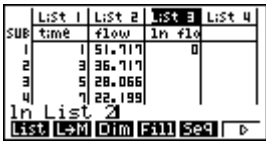
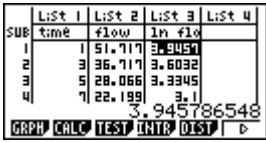
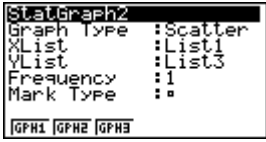
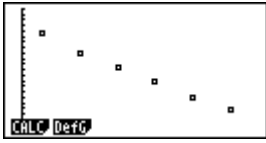
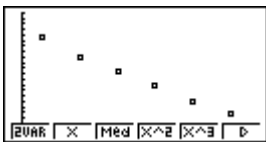
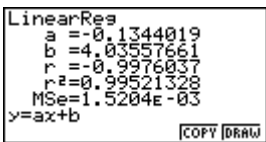
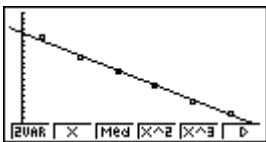
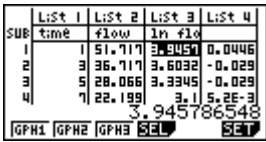
- Graphically display these data.
- Find an exponential model that best fits these data.

Please complete the linear model task before starting this task. Knowledge of the processes involved in that task is assumed here.

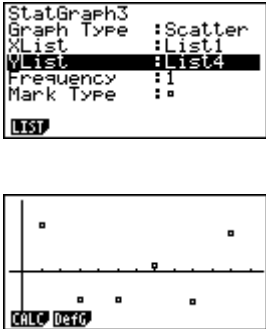
PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application ().		
Enter the SET UP ((SHIFT) then (MENU)) and set the options as seen opposite. Set the residual list to List4 and make File2 active. This way the data in File1 will not be lost. Enter the time data into List 1 and the flow data into List 2. Press (EXE) after each entry.	 	
Now select GRPH ((F1)). Press (F6) to select SET. Scroll down to adjust the StatGraph1 options as shown opposite. Press (EXE) or (EXIT) to return to the stat list window.	 	
From the stat list window, you can now make the scatterplot by pressing (F1). The scatterplot is displayed.	 	From this plot it appears that an exponential model might be an appropriate model for these data. This process is continued on the next page.

PROCESS (cont.)

ACTION	DIAGRAM	COMMENTS
To obtain an exponential model we can now try to 'straighten' the data out by finding the natural logarithm of the flow data. Put the cursor in the very top of List3. Enter In then press [OPTN], then LIST ([F1]) and then List ([F2]) followed by [2]. Press [EXE]. This will result in List3 being populated with the natural logarithms of each value in List2.	 	
We can then make a scatterplot of the natural logarithm of flow by time. To do this we have SET up StatGraph2.	 	
We can then find the linear model of best fit for these data.	  	You can now use the laws of natural logarithms to convert the following equation to the more common exponential form: $\ln f = -0.134t + 4.036$
Pressing [EXIT] twice returns us to the stat list window and we find the residual values have filled List4.		This process is continued on the next page.

PROCESS (cont.)

ACTION	DIAGRAM	COMMENTS
To help assess this model, we can make a scatterplot of the residual values vs time. We have SET up StatGraph3 to be this plot. See the settings opposite. Drawing GPH3 reveals the residual plot.		

ALGEBRAIC GENERATION OF POWER MODELS

How do I obtain, assess and use a power model?

Example:

Consider the data in the table below.

x	1	3	5	7	9	11
y	1.3	8.6	27	47.1	84	116.3

- a) Determine whether or not a power model is an appropriate model for these data.

Please complete the linear model and exponential model task before starting this task. Knowledge of the processes involved in those tasks is assumed here.

PROCESS

To complete this task we will try to 'straighten' the data out by finding the natural logarithm of both the x and y data.

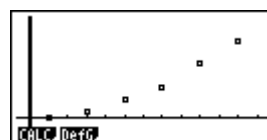
The following summarises the process. Specific detail is not provided as we have assumed you have completed the linear and exponential model task.

Firstly, we set the Stat application to use File3 (this is not necessary if you are happy to delete the data from previous tasks. We also set the residual list to List5 as we will use List3 and List 4 to compute the natural logarithm of the x and y data respectively.



After entering the data, we look at a scatterplot of y vs. x and note it is certainly non-linear.

List 1	List 2	List 3	List 4
SUB X	Y		
1	1.3		
2	8.6		
3	27		
4	47.1		
5	84		
6	116.3		

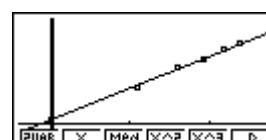
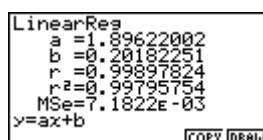
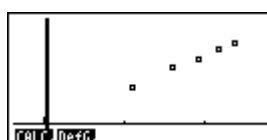
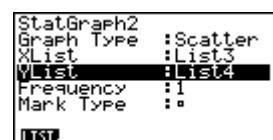


We can now proceed to try and 'straighten out' these data by computing the natural logarithms of both the x and y data.

List 1	List 2	List 3	List 4
SUB X	Y	ln X	ln Y
1	1.3	0	0.2623
2	8.6	1.0986	2.1517
3	27	1.6094	3.2958
4	47.1	1.9459	3.8522
5	84	2.1283	4.4308
6	116.3	2.4544	4.7571

List 1	List 2	List 3	List 4
SUB X	Y	ln X	ln Y
1	1.3	0	0.2623
2	8.6	1.0986	2.1517
3	27	1.6094	3.2958
4	47.1	1.9459	3.8522
5	84	2.1283	4.4308
6	116.3	2.4544	4.7571

Now, we produce a graph of ln y vs. ln x and find the linear model of best fit.



Now, using the laws of natural logarithms you can transform the following into the more traditional form of a power model:

$$\ln y = 1.90 \ln x + 0.202$$

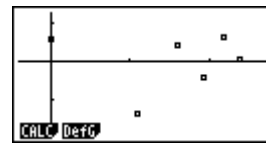
Finally, we can produce a residual plot to assist us in assessing the goodness of fit of this power model.

	List 2	List 3	List 4	List 5
SUB	Y	ln X	ln Y	
1	1.3	0	0.2623	0.0605
2	8.6	1.0986	2.1517	-0.133
3	27	1.6094	3.2958	0.0421
4	47.1	1.9459	3.8522	-0.039
			0.06054175091	
	GP1	GP2	GP3	SET

```

StatGraph3
Graph Type :Scatter
XList      :List3
YList      :List5
Frequency  :1
Mark Type  :
LIST

```



CALCULUS – DESCRIBING CHANGE.

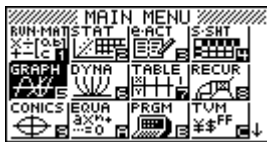
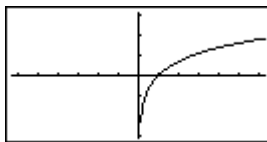
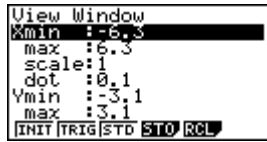
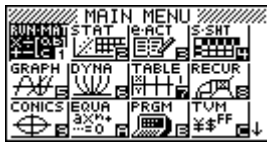
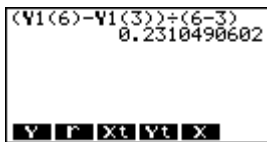
DERIVATIVES AND RELATED THINGS

How do I find the average rate of change?

Example:

If $f(x) = \ln x$, find the average rate of change of $f(x)$ for the interval $3 \leq x \leq 6$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application  .		
Define Y1 to be the function required and press EXE . Note the = sign is 'blackened' telling you this function will be graphed. Before drawing the function, you should always check your view window. Press SHIFT then F3 to check/change your settings. Press F6 to DRAW the function.	 	The INITIAL settings are often useful for 'text-book' functions. They also ensure that tracing occurs in units of 0.1 in the x direction. 
Enter the RUN application  .		
Press VAR then GRPH (F4) and then use Y (F1) and 1 to enter Y1 (which is the function we are interested in). Complete the calculation as shown opposite.		

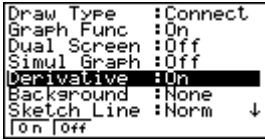
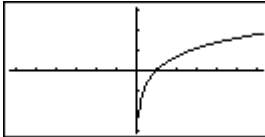
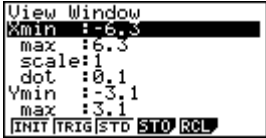
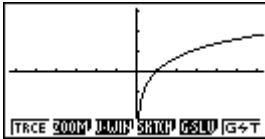
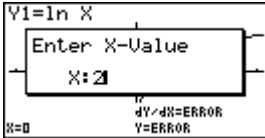
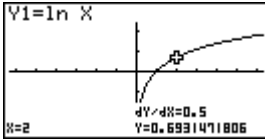
How do I find an approximation for the instantaneous rate of change using the trace function?

(Note that this process is equivalent to finding the derivative of the function at the stated point and finding the gradient of the tangent to the function at the stated point.

Example:

Find the instantaneous rate of change at $x=2$ if $y = \ln x$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application (GRAPH).		
To use the trace function to find the derivative, you will need to check the SET UP. Press SHIFT then MENU to access the SET UP options. Turn the Derivative option On (F1). Press EXE or EXIT to return to the graph editor window.		Note that when the Derivative option is turned On, some computations will take longer than when it is turned Off.
Define Y1 to be the function required and press EXE. Note the = sign is 'blackened' telling you this function will be graphed. Before drawing the function, you should always check your view window. Press SHIFT then F3 to check/change your settings. Press F6 to DRAW the function.	 	The INITIAL settings are often useful for 'text-book' functions. They also ensure that tracing occurs in units of 0.1 in the x direction. 
To activate the Trace option, press SHIFT then F1. Enter the x-value using the keypad (2 in this case) and press EXE. The derivative is displayed.	  	Note that the value computed is an approximation for the exact value. In this case, calculator displays 0.5 (which is the exact value in this case), but in some cases it may display 0.499999999. If a question requires you to find the 'exact' value, you should use the rules of algebraic differentiation to find your answer and give your answer in the appropriate form (e.g. fraction or surd).

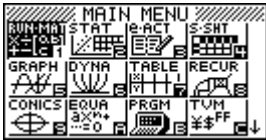
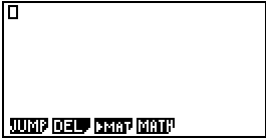
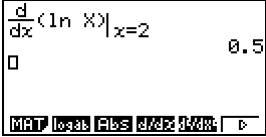
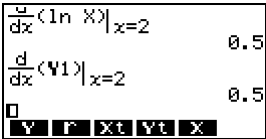
QUESTION

How do I find an approximation for the derivative at a point in RUN the application?

Example:

If $y = \ln x$, find $\frac{dy}{dx}$ at $x = 2$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the RUN application (2 1 2 1).		
Press (F4) to access the MATH options.		
Press (F4) to select the d/dx function.		
Enter the function, then arrow across and enter the x-value for which you want to evaluate the derivative. Press (EXE).		We suggest you graph the function first (see previous page). It is good practice to know what the function looks like before proceeding. This will give you a better understanding (visually) and give you a better chance at evaluating the validity of the result that is returned.
Note that if you have defined the function to be Y1 (in the Graph application) you can enter Y1 in place of the expression. In Run application, once d/dx(is entered, press (VARS) then GRPH (F4) and then use Y (F1) and (1).		

! Note that the value computed using this process is an approximation of the exact value. In this case, calculator displays 0.5 (which is the exact value in this case), but in some cases it may display 0.499999999. This is why it is termed an approximation. Should the result have been the square root of 2, the calculator would return the approximation 1.414213562

If a question requires you to find the 'exact' value, you should use the rules of algebraic differentiation to find your answer and give your answer in the appropriate form (e.g. fraction or surd)

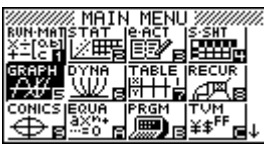
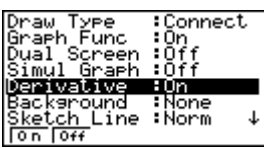
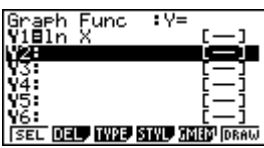
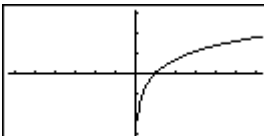
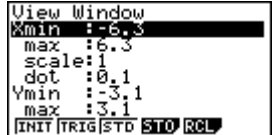
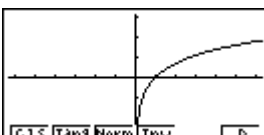
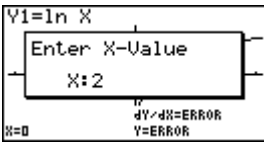
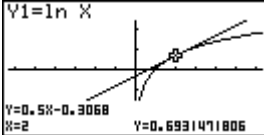
QUESTION

How do I display the tangent to a curve at a given point and find its gradient?

Example:

Find the gradient of the tangent to the curve $y = \ln x$ at $x = 2$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application  .		
To find the equation of a tangent, you will need to check the SET UP. Press [SHIFT] then [MENU] to access the SET UP options. Turn the Derivative option On ([F1]). Press [EXE] or [EXIT] to return to the graph editor window.		Note, if you do not turn the Derivative option on, the equation of the tangent/normal will not be displayed on the screen.
Define Y1 to be the function required and press [EXE] . Note the = sign is 'blackened' telling you this function will be graphed. Before drawing the function, you should always check your view window. Press [SHIFT] then [F3] to check/change your settings. Press [F6] to DRAW the function.	 	The INITIAL settings are often useful for 'text-book' functions. They also ensure that tracing occurs in units of 0.1 in the x direction. 
Press SKTCH ([SHIFT] , [F4]) to view options. Select Tang ([F2]).	 	
Enter the X-value using the keypad. You must press [EXE] twice in order for the tangent to appear. The tangent at that point will be displayed along with its equation (and therefore gradient and y-intercept).	 	Note, if you do not turn the Derivative option on, the equation of the tangent will not appear on the screen. Note that these results are approximations for the exact values. If exact values are required, used algebraic methods.

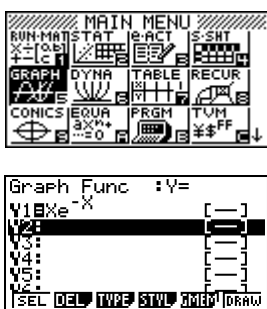
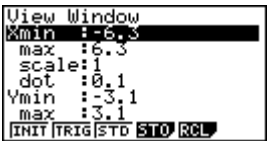
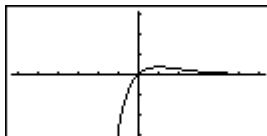
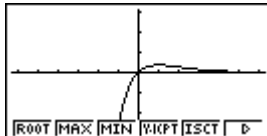
QUESTION

How do I find a numerical approximation for coordinates of the local maximum (or local minimum) of a function?

Example:

Find the coordinates of the local maximum and local minimum of $y = xe^{-x}$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application () and define Y1 to be the function required and press EXE.	 The top image shows the 'MAIN MENU' with 'GRAPH' highlighted. The bottom image shows the 'Graph Func' screen where 'Y1=xe^-x' is entered.	Before sketching the function, you should always check your view window. Press SHIFT then F3 to check/change your settings.  You can often use the zoom options (press SHIFT, F2) to help view the function. Often, Auto works well, especially if you set the domain sensibly.
Press F6 to draw the curve function.	 A graph of the function $y = xe^{-x}$ is shown on a coordinate plane. The curve starts at the origin, rises to a local maximum, and then decreases, approaching the x-axis as x increases.	It is important to note that the features you want to locate must be "in view" for the calculator to be able to locate them.
To find the coordinates of the local maximum: Press G-SOLVE (SHIFT, F5) to view options. Select MAX (F2). Use MIN (F3) to locate a local minimum (should there be one).	 The 'G-Solve' menu is shown with options: ROOT, MAX, MIN, WKPT, ISCT, and D. The 'MAX' option is highlighted.	
The calculator will display the coordinates of the local maximum (or local minimum).	 The calculator screen shows the function $Y1=Xe^{-X}$ and the coordinates of the local maximum: $X=0.9999999881$ and $Y=0.3678794412$.	If more than one local maximum (or local minimum) is in view, use the arrow keys (◀, ▶) to move to the next one.



Note that this process will not necessarily locate all of the stationary points of a function, just the ones that are visible in the view window.

To find all stationary points, or be sure that what you are seeing is all there are, you need to use algebraic methods (set the derivative equal to zero).

Also, the values computed are approximations for the exact values of the coordinates being found.

If the question requires you to find the "exact value" you should use algebraic methods.

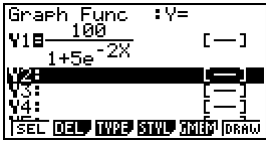
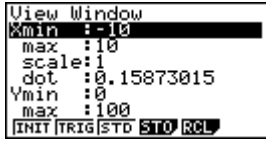
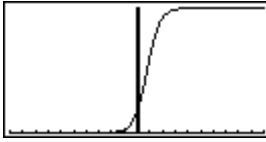
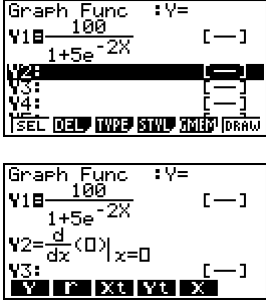
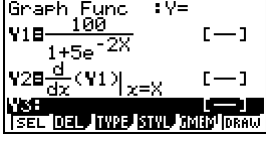
QUESTION

How do I find a numerical approximation of the coordinates of a point of inflection?

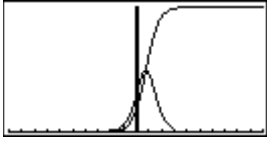
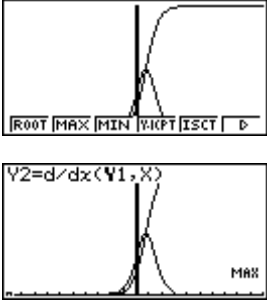
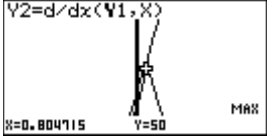
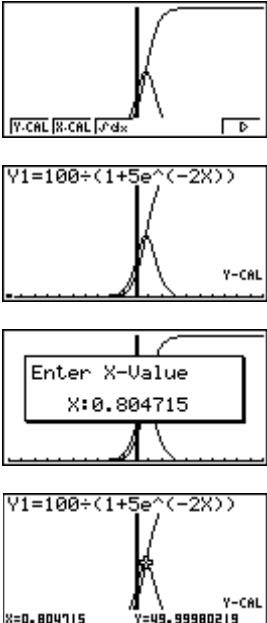
Example:

Find the coordinates of the point of inflection of $y = \frac{100}{1+5e^{-2x}}$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application (2ND Y=) and then define Y1 to be the function required and press EXE.		Before sketching the function, you should always check your view window. Press SHIFT then F3 to check/change your settings. 
Press F6 to draw the function.		You can also use the zoom options (press SHIFT, F2) to help achieve a good view of the function. Often Auto zoom works well, especially if you set the domain sensibly.
To find the point of inflection, we need to sketch the derivative function of Y1. We define Y2 to be the derivative function of Y1. Press OPTN, select CALC (F2) and then press F1 to select d/dx.		
Select Y (F1) then enter 1. Arrow across and enter X (X,0,T). Press EXE.		Continued on the next page.

Inflection (continued)

ACTION	DIAGRAM	COMMENTS
Press (F6) to draw both functions.		Functions will be graphed in order of being entered. You do not need Y1 to be graphed and so you could use the SEL ((F1)) function in the graph editor to deselect Y1. This will also speed up the process.
We need to find the turning point (local maximum in this case) of the derivative function, Y2. Press G-SOLVE ((SHIFT), (F5)) to view options. Select MAX ((F2)). Use the arrow keys ((▲), (▼)) to select the function for which the local maximum is required and press (EXE).		The x coordinate of the maximum of the derivative function gives the x coordinate of the inflection point of the function.
The coordinates of the local maximum will be displayed. This provides the x coordinate of the point of inflection of Y1.		Note that the calculator returns an approximation for the exact value of x at which the local maximum occurs.
To find the y coordinate of the point of inflection: Press G-SOLVE ((SHIFT), (F5)) to view options. Then press ((F6)) view the rest of the menu. Press Y-CAL ((F1)). Use the arrow keys ((▲), (▼)) to select the function you would like to use (Y1) and press (EXE). Enter the x-coordinate (0.804715 in this case) and press (EXE). The calculator will display the coordinates of the required point.		You could also find the y coordinate of the point of inflection in the RUN application. Press (VARS) (variable menu). Select GRPH ((F4)). Select Y (press (F1)) then enter the x-coordinate (0.804715). Close brackets and press (EXE). 

⚠ Note that this process will not necessarily locate all of the points of inflection, just the ones that are visible in the view window. Algebraic processes are required to find all points of inflection. Also, the values computed are approximations for the exact values of the coordinates being found.

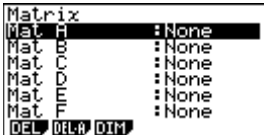
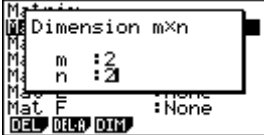
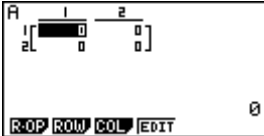
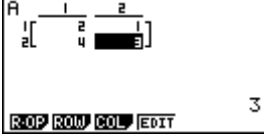
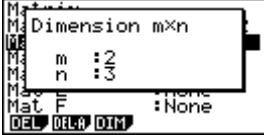
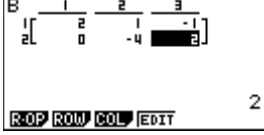
If the question requires you to find the “exact value” you should use algebraic methods.

WORKING WITH MATRICES.

MATRICES

How do I define a matrix?

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the RUN-MAT application . Press F3 to enter the MATrix mode.	 	
Select Mat A and define its size. To make it a "two by two" enter 2, press EXE then 2 and press EXE. Press EXE again to have the matrix appear.	 	
Enter the matrix elements. Press EXE after each entry.	 	If an error is made, simply highlight and overtype or press F4 to edit.
To define another matrix, press EXIT and define the size of the next matrix.	 	To begin performing matrix operations, you will need to press EXIT to return to the calculation area.

QUESTION

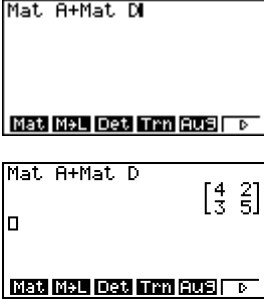
How do I perform matrix operations?

Example:

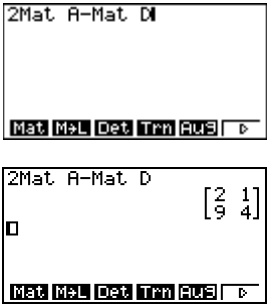
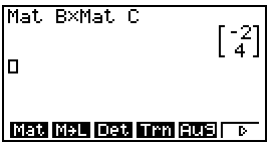
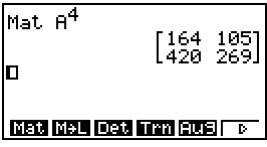
Let $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$.

Find $A + D$, BC and A^4 .

SOLUTION

ACTION	DIAGRAM	COMMENTS
The following assumes the above matrices have been defined. Enter the $\frac{\text{RUN-MAT}}{\text{2-1}} \left[\frac{\text{2-1}}{\text{2-1}} \right]$ application press OPTN and select MAT (F2).		
Various options are now available at the bottom of the screen. Use the function keys (F1 to F6) to select.		Note: the 'Mat' entry (that is so common in this topic) can also be found by pressing SHIFT then 2.
Addition $A + D$ Press Mat (F1) then ALPHA (which allows entry of the red letters) then X,0,T to enter A. Then enter + and then Mat D in a similar way. Press EXE to reveal the result matrix.		
Using and storing a result. The result matrix can be used for another calculation by using the Mat Ans command. To do so, press Mat (F1) then SHIFT then (Ans). The result matrix can also be stored for later use. Press Mat (F1) then SHIFT then (Ans) then → Mat (F1) then ALPHA followed by a letter not yet used for a matrix. Press EXE. Matrix E can now be used at will.		Note that the matrix that is recalled when Mat Ans is used is the most recently calculated result matrix.

Matrix operations continued.

ACTION	DIAGRAM	COMMENTS
Subtraction $2A - D$	 The diagram shows two calculator screens. The top screen displays '2Mat A-Mat D' and the bottom screen displays the result matrix $\begin{bmatrix} 2 & 1 \\ 9 & 4 \end{bmatrix}$.	When multiplying a matrix by a number, the multiplication sign is not needed.
Multiplication BC	 The diagram shows a calculator screen displaying 'Mat B*Mat C' and the result matrix $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$.	Note that the multiplication sign is not necessary in this calculation.
Computing a power of a matrix. A^4	 The diagram shows a calculator screen displaying 'Mat A^4' and the result matrix $\begin{bmatrix} 164 & 105 \\ 420 & 269 \end{bmatrix}$.	

LINEAR PROGRAMMING.

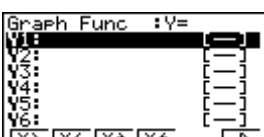
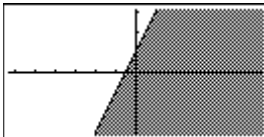
QUESTION

How do I graph an inequality?

Example:

Sketch $y \leq 2x + 1$.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application  .		
Select TYPE (F3).		
Press (F6) to view other options. Select the appropriate inequality form. In this example, select $Y \leq$ (F3).	 	
Enter the rest of the inequation and press EXE .		Note the type of inequality being graphed will appear at the top right of the calculator screen. Before sketching the function, you should always check your view window. Press SHIFT then F3 to check/change your settings. 
Press (F6) to DRAW the inequality.		

QUESTION

How do I graph a feasible region and determine the optimal solution?

Example:

a) Sketch the region bounded by the following constraints and shade the feasible region:

$$y \leq 10 - x$$

$$y \geq x - 4$$

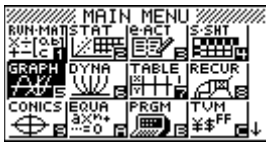
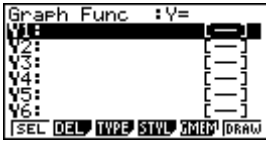
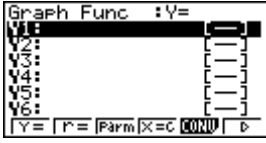
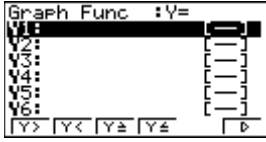
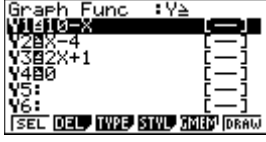
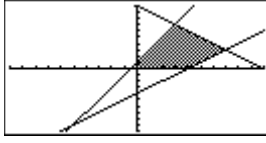
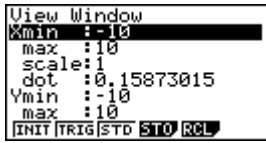
$$y \leq 2x + 1$$

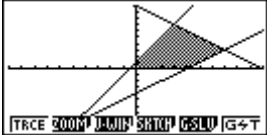
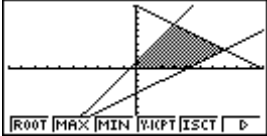
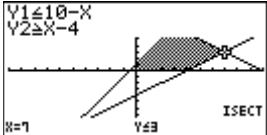
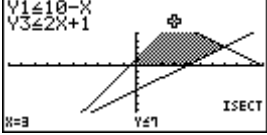
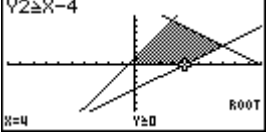
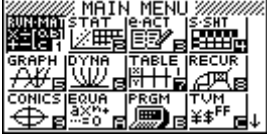
$$x \geq 0 \text{ and } y \geq 0$$

b) Determine the coordinates that are potential optimal solutions.

c) If the objective function is $P = 2x - y$, determine the optimal solution that maximises the value of P .

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application  .		
Select TYPE (F3).		
Press F6 to view other options. Select the appropriate inequality form.	 	
Enter each constraint.		
Select F6 to DRAW the feasible region.		Before sketching the function, you should always check your view window. Press SHIFT then F3 to check/change your settings. 

To determine the coordinates of the points that are potential optimal solutions (vertices of the feasible region) press G-SOLVE (SHIFT F5) to view options. Select ISCT (F5).	 	
Use the arrow keys (▲, ▼) to select the first of the pair of lines that intersect at the point you wish to find, press EXE and then select the second line and press EXE again. The coordinates of the point of intersection will be displayed at the bottom of the screen. Repeat this process until all vertices are found.	 	You may also need to find the x-intercept. Press G-SOLVE (SHIFT F5) to view options. Select ROOT (F1). Use the arrow keys (▲, ▼) to select the line for which the x-intercept is to be found and press EXE. 
To find the optimal solution, enter the RUN application (SHIFT F2).		
Substitute each of the coordinates into the objective function and compute its value.		

SOLVING EQUATIONS.

QUESTION

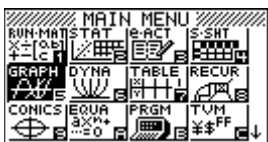
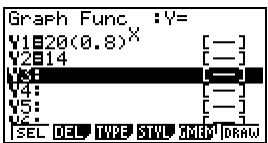
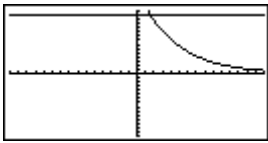
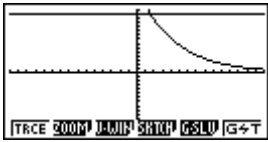
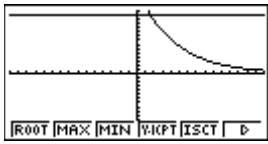
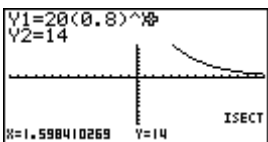
How do I solve an equation graphically?

Example:

Solve the equation:

$$20(0.8)^x = 14$$

SOLUTION

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application  .		
Define Y1 to be the left hand side of the equation and press EXE . Define Y2 to be the right hand side of the equation and press EXE .		Before sketching the functions, you should always check your view window. Press SHIFT then F3 to check/change your settings.
Press F6 (or EXE) to graph.		
To determine the solution of the equation, find the intersection point(/s) of the two functions. Press G-SOLVE (SHIFT , F5) to view options. Select ISCT (F5).	 	The calculator will find one intersection point even if it is not displayed in the graph view window. However, if there is more than one intersection point, it will not locate both unless BOTH are visible. The move from one intersection point to the other press the left or right arrow keys (◀ , ▶).
The solution will be displayed (x-coordinate).		



The values computed are approximations for the exact values of the coordinates being found.

If the question requires you to find the “exact value” you should use algebraic methods.

QUESTION

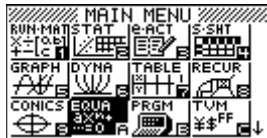
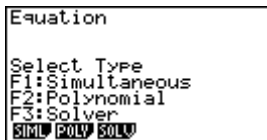
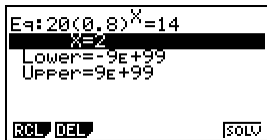
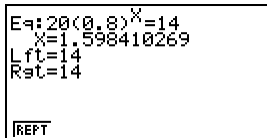
How do I solve an equation using Solver in EQUA?

Example:

Solve the equation:

$$20(0.8)^x = 14$$

SOLUTION

ACTION	DIAGRAM	COMMENTS
Enter EQUA application.		
Press (F3) to select Solver.		
Enter the equation to be solved. Press (EXE) . The value of x shown is NOT the solution, but the last value of x used by the 9860. Make certain the cursor is placed on the second line and the press Solv ((F6)).		The lower and upper bounds form the domain in which the calculator will search for a solution. You can change them as desired.
The solution will be displayed. To solve another equation, press REPT ((F1)).		We suggest that when solving equations it is wise to graph first. You will then be able to see the intersection point (solutions to the equation). The Lft and Rgt values are the values of the left and right side of the equation for the value of x found. If they are equal, then you know the solution is correct.

⚠ This process will only return one solution to the equation. There may be other solutions. You can change the first value of x that appears to a different value and another solution may be found.

Also, the values computed are approximations for the exact value of solution being found.

If the question requires you to find the “exact value” you should use algebraic methods.

QUESTION

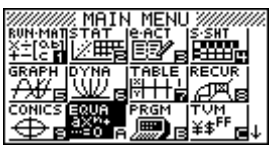
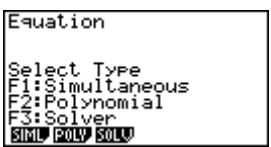
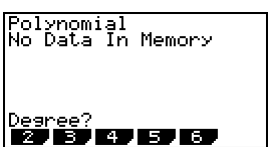
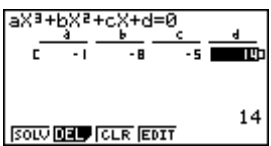
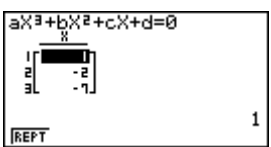
How do I find the solutions to a polynomial equation?

Example:

Solve of this equation:

$$-x^3 - 8x^2 - 5x + 14 = 0$$

SOLUTION

ACTION	DIAGRAM	COMMENTS
Enter EQUA application ()		
Press F2 to select Polynomial.		
Use the function keys, F1 or F2 , to select the degree of the polynomial, n this example, degree 3.		
Key in coefficients and constant of the equation. Press EXE after each entry. Once complete, press F1 to solve.	 	Note the form that the equation must be expressed in to determine the values of a, b, c and d for correct entry is displayed at the top of the screen.
The solution will be displayed.		Note that all solutions of the polynomial equation are given. We suggest that when solving polynomial equations it is wise to graph the polynomial first. You will then be able to see the x-intercepts (roots/solutions to the equation.)

The End.

Good luck with your exam and your future.