Do you know all there is to know about using your graphics calculator in the SACE Mathematical Studies Examination?

Specifically the CASIO fx-9860G AU PLUS 2<sup>nd</sup> Edition

Written by Elena Zema Edited by Anthony Harradine and Alastair Lupton Do you know all there is to know about using your graphics calculator in the SACE Mathematical Studies Exam? Specifically the CASIO fx-9860G AU PLUS. 2<sup>nd</sup> Edition.

Copyright © 2012 Zema, Harradine and Lupton.

Potts-Baker Institute for School Mathematics Prince Alfred College PO Box 571 Kent Town SA 5071.

### Copyright Information

The materials within, in their present form, can be used free of charge for the purpose of facilitating the learning of children in such a way that no monetary profit is made.

The materials cannot be used or reproduced in any other publications or for use in any other way without the express permission of the authors.

# CONTENTS

Linear Mode Vs Math Mode	4
WORKING WITH STATISTICS	5
Normal Distribution	
Inverse Normal	
Z-test	
Confidence Intervals  How do I find a confidence interval for the population mean?  How do I test a claim about a population proportion?  How do I find a confidence interval for the population proportion?	9 10
Binomial Distribution	
How do I calculate probabilities (non-cumulative) from a binomial distribution?	
WORKING WITH FUNCTIONS AND GRAPHS USING CALCULUS	14
Derivatives And Related Things	? . 15 16 17 cal 19
Integration	?. 22 23
WORKING WITH LINEAR EQUATIONS AND MATRICES	
Matrices  How do I define a matrix?  How do I perform matrix operations?  Addition  Using and storing a result.  Subtraction  Multiplication  Computing a power of a matrix.  Inverse  Determinant	27 28 28 28 29 29 29
Systems of Linear Equations	30 31
POLYNOMIAL EQUATIONS.	
Solving Polynomial Equations	34
HOW NO FIND THE SOLUTIONS TO A POLYHOLLING EMPARIOLE	J4

# LINEAR MODE VS MATH MODE

The 9860 has two operating modes: Linear Mode and Math Mode.

Linear Mode is the traditional mode of scientific calculators where calculations are entered in a line, as seen opposite.

J(e^(X),1,2)

Sour Eners Swar from South | D

Math Mode allows the user to work with calculation entry that looks 'natural'. This is often termed natural input. It can be seen opposite.



To set your 9860 to work in the mode you desire, enter the SET UP when in the RUN•MAT (ﷺ) application. To do this press ∰ then ∰ You can then choose the option you want.



Math Mode is the default setting for 9860's using OS version 2.0 and later.

In this book all instructions assume you have your 9860 set to work in Math Mode.

# **WORKING WITH STATISTICS**

# NORMAL DISTRIBUTION

# How do I find probabilities from normal distributions? Example:

A certain brand and flavour of chip is sold in bags stamped as containing 125 grams. The chips are delivered to each bag by a machine and the weight varies from packet to packet. Let W denote the weight of the chips in a packet. The distribution W can be modeled by a normal distribution with a mean of 125.8 grams and a standard deviation of 0.37 grams.

Determine the proportion of packets that will contain less than 125 grams of chips.

#### PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  (	SUB    CREE   List 2 List 3 List 4	
Press [F1] to access the NORMal distribution menu.	LIST I LIST E LIST E LIST U SUB I E E E E E E E E E E E E E E E E E E	
Press <b>F2</b> to access NCD (normal cumulative distribution) function.	SUB   LiSt 1 LiSt 2 LiSt 3 LiSt 4	
Input the values. Press EXE after each entry.	Normal C.D  Data :Variable Lower : 0  Upper : 0  Save Res: None  List  var   Normal C.D Lower : -1E+99 ↑ Upper : 125  0 : 0,37  P : 125.8 Save Res: None	In this example, we want to find P(W<125).  The lower limit of -1E99 is used to reflect the characteristics of the normal distribution.  To find P(120 <w<125) 120="" 125.<="" and="" lower="" set="" td="" to="" upper=""></w<125)>
Once complete, press [F1] (or [XE]) to CALCulate the probability. The probability will be displayed along with the corresponding Z scores.	Normal C.D P =0.01530283 z:Low=-2.703e+99 z:UP =-2.1621622	Therefore, P(W<125) = 0.0153 (or 1.53%) - correct to 3 significant figures.
Alternatively, press <b>F6</b> to DRAW the normal distribution and shade the region of interest, as well as display the probability along with the corresponding Z scores.	zilow=-ze99 ziUp=-z.162 P=0.0159028994	

Note that, when using some older operating systems, the use of -1 E 99 or 1 E 99 as the lower or upper limit may at times result in the calculator returning Ma Error. Should this occur, use -1 E 90 or 1 E 90 (or a similar suitably small or large number).

# INVERSE NORMAL

# How do I find a score, given a normal probability value? Example:

The height of all year 12 female students is assumed to be normally distributed with a mean of 175 cm and a standard deviation of 6 cm.

Find how tall Elena is if she is taller than 95% of year 12 students?

ACTION	DIAGRAM	COMMENTS
Enter the STAT application ([]]).	MAIN MENU SHIT SHIT SHIT SHIT SHIT SHIT SHIT SHIT	
Press <b>F5</b> to access the DISTribution menu.	LIST I LIST 2 LIST 3 LIST 4  SUB  2  3  4  MRPD CALC TEST UNITE DIST	
Press <b>F1</b> to access the NORMal distribution menu.	LIST I LIST 2 LIST 3 LIST U SUB I 2 3 4 NORM t CHI F BINM D	
Press [F3] to access InvN (inverse normal) function.	SUB LIST I LIST E LIST B LIST U  E B B B B B B B B B B B B B B B B B B	
Input the values. Press EXE after each entry.  Once complete, press F1 (or EXE) to calculate the value of the score.	Inverse Normal Tail :Left Area :0.95 6 :6 P :175 Save Res:None	In this example, we want to find x such that $P(X < x) = 0.95$ . So the area we know is to the left of the unknown score. Hence we set the "Tail" setting to Left. Had we wanted to compute $P(X > x) = 0.2$ , we would enter 0.2 for the area and set the "Tail" setting to Right.
The value of the score will be displayed.	Inverse Normal %Inv=184.869122	Therefore, x = 185 cm correct to 3 significant figures.

# Z-TEST

# How do I carry out the calculations for a Z-test?

#### Example:

A machine that delivers fruit to a can is known to deliver portions with mean 375 grams with a standard deviation of 14.2 grams. Changes are made to the machine. Following the changes, a random sample of 100 portions is taken and the average weight of this sample is found to be 379 grams.

Use a two-tailed Z-test, at the 5% level of significance, to determine whether or not the mean weight of the portions is now different to 375 grams. Assume the standard deviation has not changed.

PROCESS	DIAGRAM	0014451470
ACTION	DIAGRAM	COMMENTS
Enter the STAT application  ( ) and then press F3 to access the TEST menu.	SUB   List   List 2 List 3 List 4	
Press <b>F1</b> to access the Z menu.	LIST I LIST 2 LIST 3 LIST 4 SUB I I I I I I I I I I I I I I I I I I I	
Press <b>F1</b> to access the 1-S (one sample) function.	LiSt   LiSt	
Press F2 to select VARiable.  Input the values. Press EXE after each entry.  Once complete, press F1 (or EXE) to have the calculations carried out.	1-Sample ZTest	Note that a two-tailed test is required for the Mathematical Studies course, so set $\mu \neq \mu 0$ .  Also note that the standard deviation of the population is entered, not the standard error.  If you press <b>F6</b> DRAW, the distribution will be drawn and the Z score and p value given at the bottom of the view window.
The results are displayed.	1-Sample ZTest µ ≠375 z =2.81690141 p =4.8489e-03 \$\overline{x}\$ =379 n =100	In this case, since the p value is less than 0.05, sufficient evidence exists to reject the null hypothesis.

# CONFIDENCE INTERVALS

How do I find a confidence interval for the population mean? Example:

In 1990 a study was undertaken in which the body temperature of 130 healthy men from one nation were measured. The mean temperature of the sample was found to be 36.8  $^{\circ}$ C. Let  $\mu$  be the mean body temperature of all healthy men from this particular nation in 1990.

Assuming the population standard deviation is 0.5  $^{\circ}$ C, find a 95% confidence interval for  $\mu$ . PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the STAT application (	MAIN MENU MENU MENU MENU MENU MENU MENU MEN	
Press F4 to access the INTRerval menu.	SUB LIST I LIST Z LIST 3 LIST U Z 3 U MRPH CALC TEST UNITS DIST	
Press F1 to access the Z menu.	List   List	
Press [F1] to access the 1-S (one sample) function.	LiSt   LiSt 2 LiSt 3 LiSt 4	
Press <b>F2</b> to select VARiable.	1-Sample ZInterval    Data	
Input the values. Press <b>EXE</b> after each entry.  Once complete, press <b>F1</b> (or <b>EXE</b> ) to calculate the interval.	1-Sample ZInterval C-Level:0.95 ↑ 0 :0.5 \$\frac{2}{2}\$ :36.8 n :130 Save Res:None	Note that the standard deviation of the population is entered, <b>not</b> the standard error.
The confidence interval will be displayed.	1-Sample Zinterval Left =36.7140499 Risht=36.8859501 Z =36.8 n =130	

# How do I test a claim about a population proportion?

Example:

The Gawler council decides to close Junction Road. A random sample of 401 residents from Gawler was asked whether they thought the road should stay open. 212 of the 401 surveyed said the road should stay open to traffic.

Is there sufficient evidence to suggest that the proportion of all Gawler residents who thought the road should stay open is different to 50%?

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  (	SUB   LIST   LIST 2 LIST 3 LIST 4	
Press [F1] to access the Z menu.	LIST I LIST 2 LIST 3 LIST 4	
Press <b>F3</b> to access the 1-P (one proportion) function.	LiSt   LiSt	
Input the values. Press EXE after each entry.	1-Prop ZTest Prop :≠pø PØ :0 x :0 n :0 Save Res:None Execute [≠ [< [>>	Prop is the sample proportion test conditions (≠p specifies two-tail test, which is required in Mathematical Studies). p0 is the null proportion. x is the number of 'successes' in the sample. n is the size of the sample.
Once complete, press <b>F1</b> (or <b>EXE</b> ) to calculate the results.	1-Prop ZTest Prop :≠p0 p0 :05 x :212 n :401 Save Res:None  axeguia   Calc	Press F6 to display the results graphically.  Press F1 and F2 to display
The results will be displayed.	1-Prop ZTest Prop≠0.5 z =1.14856519 p =0.25073531 p̂ =0.5286783 n =401	the Z and p value respectively.  In this case, since the p value is greater than 0.05, we do not reject the null hypothesis.

# How do I find a confidence interval for the population proportion?

#### Example:

The Gawler council decides to close Junction Road. A random sample of 401 residents from Gawler was asked whether they thought the road should stay open. 212 of the 401 surveyed said the road should stay open to traffic.

Calculate an approximate 95% confidence interval for the proportion of all Gawler residents who thought the road should stay open.

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  (Example 1) then press F4 to access the INTRerval option.	SUB   List   List   List   List	
Press F1 to access the Z menu.	SUB I I I I I I I I I I I I I I I I I I I	
Press <b>F3</b> to access the 1-P (one proportion) function.	SUB   L:St   L:St   L:St     L:St	
Input the values. Press <b>EXE</b> after each entry.	1-Prop ZInterval  D=Level	
Once complete, press <b>F1</b> (or <b>EXE</b> ) to calculate the interval.	1-Prop ZInterval C-Level :0,95 x :212 n :401 Save Res: None Lacoute	Note that x is the number of "successes" in the sample (supporters in this case).
The confidence interval will be displayed.	1-Prop ZInterval Left =0.4798209 Right=0.5775357 \$ =0.5286783 n =401	

# BINOMIAL DISTRIBUTION

# How do I calculate probabilities (non-cumulative) from a binomial distribution?

#### Example:

The label on each packet of a certain brand and flavour of chip states each bag contains 125 grams of chips. Suppose that 1.5% of all bags produced contain less than 125 grams of chips. Let Y be the number of packets in the random sample that contain less than 125 grams of chips.

A random sample of 5 packets is selected. Use the binomial distribution to find the probability that 2 packets will contain less than 125 grams of chips?

ACTION	DIAGRAM	COMMENTS
Enter the STAT application (Example and then press F5 to access the distribution menu.	SUB  I  I  I  I  I  I  I  I  I  I  I  I  I	
Press <b>F5</b> to access the BINMomial distribution.	LIST I LIST E LIST E LIST U  SUB  I  E  U  NORM CHI F BINM D	
Press <b>F1</b> to access Bpd (binomial point distribution) function.	L:St   L:St 2 L:St 3 L:St 4	
Press <b>F2</b> to select VARiable.	Binomial P.D  Data :Variable  x :0  Numtrial:0  P :0  Save Res:None  Execute [List   Var	
Input the values. Press EXE after each entry.  Once complete, press F1 (or EXE) to calculate the probability.	Binomial P.D Data :Variable x x X Numtrial:5 P :0.015 Save Res:None   xecute	In this example, we want to find P(Y=2), where Y can be modeled by B(5, 0.015).
The probability will be displayed.	Binomial P.D P=2.1503e-03	Therefore, P(Y=2) = 0.00215 or 0.215% (3 significant figures).

# How do I calculate probabilities (cumulative) from a binomial distribution?

#### Example:

The label on each packet of a certain brand and flavour of chip states each bag contains 125 grams of chips. Suppose that 1.5% of all bags produced contain less than 125 grams of chips. Let Y be the number of packets in the random sample that contain less than 125 grams of chips.

A random sample of 20 packets is selected. Use the binomial distribution to find the probability that more than 2 packets will contain less than 125 grams of chips?

ACTION	DIAGRAM	COMMENTS
Enter the STAT application  ( ) and then press F5 to access the distribution menu.	SUB   L:St   L:St   L:St   L:St   U	
Press <b>F5</b> to access the BINMomial distribution.	SUB LIST I LIST E LIST E LIST U SUB I I I I I I I I I I I I I I I I I I I	
Press <b>F2</b> to access Bcd (binomial cumulative distribution) function.	LiSt   LiSt 2 LiSt 3 LiSt 4	
Press [F2] to select VARiable.	Binomial C.D  Data :Variable  X  Numtrial:0 P Save Res:None Execute [List  Var	
Input the values. Press EXE after each entry.  Once complete, press F1 (or EXE) to calculate the probability.	Binomial C.D Data :Variable x :2 Numtrial:20 P :0.015 Save Res:None	The Bcd function only calculates "less than or equal to" probabilities.  In this example, we want to find P(Y>2), where Y can be modeled by B(5, 0.015).  So first we must compute P(Y≤2) and then subtract from 1.
We can now compute 1- P(Y≤2).	1-0.99682191 0.00317809 0	

# WORKING WITH FUNCTIONS AND GRAPHS USING CALCULUS.

# DERIVATIVES AND RELATED THINGS

How do I find an approximation for the derivative at a point using the trace function?

Example:

If  $y = \ln x$ , find  $\frac{dy}{dx}$  at x = 2.

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application ( ABP).	MAIN MENU  BUHMMISTAT PACT SSHT  \$100 UEEE EEU SETT  GRAPH DYNA TABLE RECUR  ONICS EQUA FROM TUM  CONICS EQUA FROM TUM  B \$20 R B \$4 FF B \$1	
To use the trace function to find the derivative, you will need to check the SET UP.  Press SHIFT then MENU to access the SET UP options.  Turn the Derivative option On (F1).  Press EXE or EXIT to return to the graph editor window.	Input/Output:Math Draw Type :Connect Ineq Type :And Graph Func :On Dual Screen :Off Simul Graph :Off Derivative :On Un  Off	Note that when the Derivative option is turned On, some computations will take longer than when it is turned Off.
Define Y1 to be the function required and press [XE]. Note the = sign is 'blackened' telling you this function will be graphed.  Before drawing the function, you should always check your view window. Press [SHIFT] then [F3] to check/change your settings.  Press [F6] to DRAW the function.	Graph Func : Y= V181n X [-] V3: V4: V5: V6: V6: V6: V6: V6: V6: V6: V6: V6: V6	The INITial settings are often useful for 'text-book' functions. They also ensure that tracing occurs in units of 0.1 in the x direction.  Uiew Window Main 1-6.3  max 16.3  scale:1  dot :0.1  Ymin -3.1  Max 3.1  INITITIES STO SEE
To activate the Trace option, press SHFT then F1.  Enter the X-value using the keypad and press EXE. The cursor will 'jump' to the required point.  The derivative is displayed.	TRCE	Note that the value computed is an approximation for the exact value. In this case, calculator displays 0.5 (which is the exact value), but in some cases it may display 0.499999999. This is why it is termed an approximation.  If a question requires you to find the 'exact' value, you should use the rules of algebraic differentiation to find your answer and give your answer in the appropriate form (e.g. fraction or surd)

How do I find an approximation for the derivative at a point in RUN the application?

Example:

If 
$$y = \ln x$$
, find  $\frac{dy}{dx}$  at  $x = 2$ .

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the RUN application	MAIN MENU /// MA	
Press <b>F4</b> to access the MATH options.	SUMP DELY PHAT MAIP	
Press F4 to select the d/dx function.	MATE TOTAL FIRS SAME SAME:	
Enter the function, then arrow across and enter the x-value for which you want to evaluate the derivative. Press EXE.	d	We suggest you graph the function first (see previous page). It is good practice to know what the function looks like before proceeding. This will give you a better understanding (visually) and give you a better chance at evaluating the validity of the result that is returned.
Note that if you have defined the function to be Y1 (in the Graph application) you can enter Y1 in place of the expression.  In Run application, once d/dx( is entered, press (F1) and then use Y (F1) and 1.	$\frac{\frac{d}{dx}(\ln X) _{x=2}}{\frac{d}{dx}(Y^{1}) _{x=2}} = 0.5$	

Note that the value computed using this process is an approximation of the exact value. In this case, calculator displays 0.5 (which is the exact value in this case), but in some cases it may display 0.499999999. This is why it is termed an approximation. Should the result have been the square root of 2, the calculator would return the approximation 1.414213562

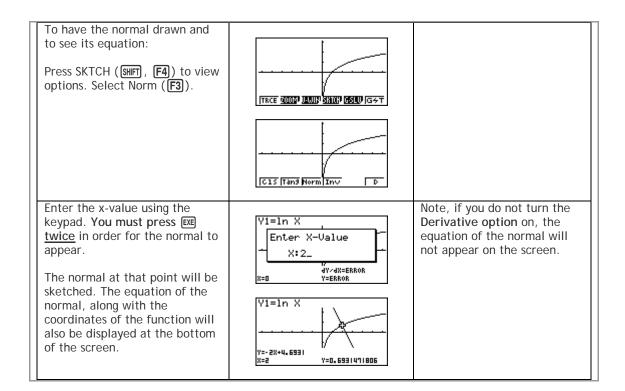
If a question requires you to find the 'exact' value, you should use the rules of algebraic differentiation to find your answer and give your answer in the appropriate form (e.g. fraction or surd)

How do I find the equation of the tangent and normal to a curve at a given point?

# Example:

Find the equation of the tangent and normal to the curve  $y = \ln x$  at x = 2.

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application ( ).	MAIN MENU WILLIAM STAT PART SANT PART PART PART PART PART PART PART PAR	
To find the equation of a tangent, you will need to check the option settings in SET UP. Press SHIFT then MENU to access the SET UP options.  Turn the Derivative option On (F1).  Press EXE or EXIT to return to the graph editor window.	Input/Output:Math Draw Type :Connect Ineq Type :And Graph Func :On Dual Screen :Off Simul Graph :Off Derivative :On U	Note, if you do not turn the Derivative option on, the equation of the tangent/normal will not be displayed on the screen.
Define Y1 to be the function required and press [XE]. Note the = sign is 'blackened' telling you this function will be graphed.  Before drawing the function, you should always check your view window. Press [SHFT] then [F3] to check/change your settings.  Press [F6] to DRAW the function.	Graph Func :Y= V181n X  V3: V4: V5: V6: [-] [SEL DEL MAR SIMU MIN DRAW	The INITial settings are often useful for 'text-book' functions. They also ensure that tracing occurs in units of 0.1 in the x direction.  View Window  XMIN 1-6.3  Scale:1  dot:0.1  Ymin:-3.1  max:3.1  INITITRIGISTO STO ROL
Press SKTCH (SHIFT), F4) to view options. Select Tang (F2).	TRCE 2001 IN SHIP SHIP GST	
Enter the x-value using the keypad. You must press EXE twice in order for the tangent to appear.  The tangent at that point will be drawn. The equation of the tangent, along with the coordinates of the function will be displayed at the bottom of the screen.	Y1=ln X Enter X-Value X:2  W=0 Y=ERROR  Y=0.5X-0.3068 Y=0.693 41 806	Note, if you do not turn the Derivative option on, the equation of the tangent will not appear on the screen.  Note that these results are approximations for the exact values. If exact values are required, used algebraic methods.



How do I find a numerical approximation for coordinates of the local maximum (or local minimum) of a function?

#### Example

Find the coordinates of the local maximum and local minimum of y = (x+1)(x-1)(x-3).

#### **PROCESS**

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application ( ) and define Y1 to be the function required and press EXE.	MAIN MENU MAIN MENU MAIN MENU MENU MENU MENU MENU MENU MENU MEN	Before sketching the function, you should always check your view window.  Press SHFT then F3 to check/change your settings.  View Window MMIN = 6.3  max = 6.3  scale : 1  dot : 0.1  Ymin = -3.1  INITITIZESTO SID [20]  You can often use the zoom options (press SHFT, F2) to help view the function.  Often, Auto works well, especially if you set the domain sensibly.
Press F6 to draw the curve function.		It is important to note that the features you want to locate must be "in view" for the calculator to be able to locate them.
To find the coordinates of the local maximum:  Press G-SOLVE (SHFT, F5) to view options. Select MAX (F2).  Use MIN (F3) to locate the local minimum.	ROOT MAX MIN WICHT ISCT D	
The calculator will display the coordinates of the local maximum (or local minimum).	V1=(X+1)(X91)(X-3)  dy/dx=0 X=-0.1541005113 Y=3.019201436	If more than one local maximum (or local minimum) is in view, use the arrow keys ( , ) to move to the next.

Note that this process will not necessarily locate all of the stationary points of a function, just the ones that are visible in the view window.

<u>To find all stationary points</u>, or be sure that what you are seeing is all there are, you need to use algebraic methods (set the derivative equal to zero).

Also, the values computed are approximations for the exact values of the coordinates being found. If the question requires you to find the "exact value" you should use algebraic methods.

How do I find a numerical approximation for the coordinates of a point of inflection?

Example:

Find the coordinates of the point of inflection of  $y = \frac{100}{1 + e^3 e^{-x}}$  for  $0 \le x \le 10$ .

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application ( ) and then define Y1 to be the function required and press [XE].	Graph Func :Y= V1= 100 V1= 100 1+e <sup>3</sup> e <sup>-X</sup> V3: [-] V4: [-] V4: [-]	Before sketching the function, you should always check your view window. Press SHFT then F3 to check/change your settings.  View Window Main 18 Max: 12.6 Scale: 1 dot: 0.1 Ymin: 0 Max: 100 [INIT   TRIG   STO   RCL.]
Press F6 to draw the function.		You can also use the zoom options (press SHFT), F2) to help achieve a good view of the function. Often Auto zoom works well, especially if you set the domain sensibly.
To find the point of inflection, we need to sketch the derivative function of Y1. We define Y2 to be the derivative function of Y1.  Press OPTN, select CALC (F2) and then press F1 to select d/dx.	Graph Func :Y= $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	
Select Y ( <b>F1</b> ) then enter <b>1</b> . Arrow across and enter X (瓜町). Press <b>EXE</b> .	Graph Func : Y=  V18 100  1+e <sup>3</sup> e <sup>-X</sup> [-]  V28 d/dx (Y1)  <sub>X=X</sub> [-]  V31  ISEL	Continued on the next page.

Inflection (continu	ed)	
ACTION	DIAGRAM	COMMENTS
Press F6 to draw both functions.		Functions will be graphed in the order of entry.  You do not need Y1 to be graphed and so you could use SEL (F1) in the graph editor to deselect Y1. This will also speed up the process.
We need to find the turning point (local maximum in this case) of the derivative function, Y2.  Press G-SOLVE (SHF), F5) to view options. Select MAX (F2).  Use the arrow keys ( , ) to select the function for which the local maximum is required and press EXE.	ROOT MAX MIN WIGHT ISCT D  Y2=d/dx(Y1,X)  MAX	The x coordinate of the maximum of the derivative function gives the x coordinate of the inflection point of the function.
The coordinates of the local maximum will be displayed.  This provides the x coordinate of the point of inflection of Y1.	V2=d/dx(V1,X)  dy/dx=d  y=25  MAX	
To find the y coordinate of the point of inflection: Press G-SOLVE (SHFT, F5) to view options.  Then press (F6) view the rest of the menu. Press Y-CAL (F1).  Use the arrow keys (A, V) to select the function you would like to use (Y1) and press EXE.	V-Cal (X-Cal (X-Ca) (X-	You could also find the y-coordinate of the point of inflection in the RUN application.  Press MARS (variable menu).  Select GRPH (F4).  Enter Y1 (press F1 1) then enter the x-coordinate in brackets(3 in this case).  Press EXE.
Enter the x-coordinate (3 in this case) and press ExE.  The calculator will display the coordinates of the required point.	Y1=100÷(1+e^(3)e^(-X)	V1(3) 50

Note that this process will not necessarily locate all of the points of inflection, just the ones that are visible in the view window. Algebraic processes are required to find all points of inflection. Also, the values computed are approximations for the exact values of the coordinates being found.

If the question requires you to find the "exact value" you should use algebraic methods.

# INTEGRATION

How do I find an approximation for a numerical integral using the GRAPH application?

Example:

Find 
$$\int_4^6 (220xe^{-0.4x} + 15) dx$$
.

PROCESS

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application and then define Y1 to be the function required; press	Graph Func :Y= V1=220Xe <sup>-0.4X</sup> +15 V2:	Before sketching the function, you should always check your view window. Press SHFT then F3 to check/change your settings.  View Window Man 18
Press [F6] to draw the curve function.		You can often use the zoom options (press SHFT), F2) to help view the function. Often, Auto works well, especially if you set the domain sensibly.
Press G-SOLVE (SHIFT), (F5) to view options. Press (F6) to see the rest of the options.  Press F3 to select $\int dx$ .	Y-CAL N-CAL IV-dx	
The calculator will prompt you to select the lower value. Rather than using the arrow pad to move the cursor to the approximate lower value, enter the Lower Bound using the keypad (4 in this case) and press [XE].	Y1=220X(e^(-0.4X))+15  LOWER-  Y1=220Xe^(-0.4X)+15	
It will then prompt you to select the upper value. Enter the Upper Bound (6 in this case) and press EXE.	Y1=220Xp^(-0.4X)+15   Enter Upper Bound   X:6	
The integral will be displayed.	LOWER=4 UPPER=5 /dx=321, 6136202	

The value computed is an approximation for the exact value of the integral. If the question requires you to find the "exact value" you should use algebraic methods.

How do I find an approximation for a numerical integral using the RUN application?

Example:

Find 
$$\int_{4}^{6} (220xe^{-0.4x} + 15) dx$$
.

# PROCESS

ACTION	DIAGRAM	COMMENTS
Enter RUN application ( and then press MATH( A), then F6 to see more options.	□ D	
Press <b>F1</b> to select $\int dx$ .	John ≥( D	
Enter the function, followed by the lower and upper bounds, using the arrow pad. Press [XE].	∫6220Xe <sup>-0.4X</sup> +15dx 327.6736202 □ 327.6736202	We suggest you graph the function first (see previous page). It is good practice to know what the function looks like before proceeding. This will give you a better understanding (visually) and give you a better chance at evaluating the validity of the result that is returned.
If Y1 is defined as the function you require (in the Graph application), you can proceed as follows: When entering the function, press WARS (variable menu). Select GRPH (F4).	Odx  Duit fact sint cast own to	
Select Y (F1) then enter 1, followed by the lower and upper limits using the arrow pad.  Press EXE.	∫_0dx y r xt yt x	
	6 4 4 327.6736202 0 Y P Xt Yt X	

When using this process the value computed is an approximation for the exact value of the integral. If the question requires you to find the "exact value" you should use algebraic methods.

# How do I find the area between two functions?

Example

Find the area enclosed by the graphs of  $y = 1 - x^2$  and y = 1 - x.

ACTION	DIAGRAM	COMMENTS
Enter the GRAPH application, ( Art and Y2 as the functions required.	Graph Func :Y= V181-X <sup>2</sup> [—] V281-X [—] V4: [—] V5: [—]   SEL   GLE   SWID   MILE   DRAW	Before sketching the function, you should always check your view window. Press SHFT then  F3 to check/change your settings.  View Window Man 1-6.3  max 16.3  scale:1 dot :0.1  Ymin :-3.1  Max :3.1  INITITRIGISTO STO RCL
Press F6 to draw the graphs.		You can also use the zoom options (press SHFT, F2) to help view the function. BOX works well if you want to focus on a specific rejoin of the graph.
To find the points of intersection, press GSOLV (SHFT, F5), then ISCT (F5).  The calculator will find the intersection point of the two functions and display the coordinates.  If more intersection points are visible, use the arrow keys to move onto them and have the coordinates displayed.	V1=1-X2 V2=1-X V2=1-X V2=1-X V2=1-X V2=1-X V2=1-X V2=1-X	Note that the calculator will only find points of intersection that are visible on the screen.
Enter RUN application ( $\P$ ) and then press MATH( $\P$ 4), then $\P$ 6 to see more options. Press $\P$ 1 to select $\int dx$ .  As Y1 and Y2 are defined to be the functions we are using, proceed as follows: Press $\P$ 8 (variable menu). Select GRPH ( $\P$ 4).	Odx St D	Process continued on the next page.

#### Area between two functions continued.

ACTION	DIAGRAM	COMMENTS
Select Y (F1) 1 Y(F1)  2, followed by the lower and upper limits using the arrow pad.  Press EXE.	$ \int_{0}^{1} V1 - V2 dx $ $ \frac{1}{6} $ $ V                                  $	
Alternatively you could enter the functions instead of using Y1 and Y2.	$ \begin{bmatrix} \frac{1}{6} & 1 - x^2 - (1 - x) dx \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} $	Note the careful use of brackets here.

When using this process the value computed is an approximation for the exact value of the area required.

If the question requires you to find the "exact value" you should use algebraic methods.

# WORKING WITH LINEAR EQUATIONS AND MATRICES.

# MATRICES

# How do I define a matrix?

ACTION	DIAGRAM	COMMENTS
Enter the RUN-MAT application ( ).  Press F3 to enter the MATrix mode.	MAIN MENU	
	SUMPICEL I PMAT MATH	
Select Mat A and define its size. To make it a "two by two" enter 2, press EXE then 2 and press EXE. Press EXE again to have the matrix appear.	Matrix Wat H None Mat B None Mat C None Mat C None Mat E None Mat E None Mat E None Mat E None	
	Material Mat	
Enter the matrix elements.  Press EXE after each entry.	ROP ROW GOL EDIT	If an error is made, simply highlight and overtype or press F4 to edit.
	ROP ROW GOL EDIT	
To define another matrix, press <b>EXIT</b> and define the size of the next matrix.	Material Mat	To begin performing matrix operations, you will need to press EXIT to return to the calculation area.
	B 1 2 3 1 1 1 2 2 3 3 3 3 3 3 3 3 3 3 3 3	

How do I perform matrix operations?

Example:

Let 
$$A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$
,  $B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -4 & 2 \end{bmatrix}$ ,  $C = \begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$  and  $D = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ .

Find A+D , BC ,  $A^4$  ,  $A^{-1}$  and  $\det A$  .

# SOLUTION

ACTION	DIAGRAM	COMMENTS
The following assumes the above matrices have been defined. Enter the steel application press OPTN and select MAT (F2).		
Various options are now available at the bottom of the screen. Use the function keys (F1 to F6) to select.	O Mat MAL Det Trn Aus o	Note: the 'Mat' entry (that is so common in this topic) can also be found by pressing SHFT then 2.
$\begin{array}{c} \textbf{Addition} \\ A+D \end{array}$	Mat A+Mat DN	
Press Mat (F1) then ALPHA (which allows entry of the red letters) then (X.6.1) to enter A.	Mat M+L Det Trm Aus D  Mat A+Mat D  [4 2] [3 5]	
Then enter  and then Mat D in a similar way.  Press  to reveal the result matrix.	Mat Mal Det Trn Gus D	
Using and storing a result.  The result matrix can be used for another calculation by using the Mat Ans command.	Mat A+Mat D Done 2×Mat Ansl  Mat M+L Det Frm Aus D	Note that the matrix that is recalled when Mat Ans is used is the most recently calculated result matrix.
To do so, press Mat (F1) then SHFT then (-) (Ans).  The result matrix can also be stored for later use.  Press Mat (F1) then SHFT then	Mat   MAT	
(Ans) then → Mat (F1) then (ALPHA) followed by a letter not yet used for a matrix.  Press EXE.  Matrix E can now be used at will.	ZXMAL HNS [8 4] 6 10] Mat Ans>Mat E [8 4] 6 10] O Mat NAL COL Trn GUS D	

Matrix operations continued.		
ACTION	DIAGRAM	COMMENTS
Subtraction $2A-D$	2Mat A-Mat DI	When multiplying a matrix by a number, the multiplication sign is not needed.
	Mat Mat Det Trn Gus D  2Mat A-Mat D  [2 1] [9 4]  [0   Mat Mat Det Trn Gus D	
Multiplication BC	Mat B×Mat C  [-2]  [-2]  [Mat Mat Oct Trn Gus D	Note that the multiplication sign is not necessary in this calculation.
Computing a power of a matrix. $A^4$	Mat A <sup>4</sup> [164 105] [420 269]  Mat Mat Det Trn Gus D	
Inverse $A^{-1}$ To enter the inverse, use the $x^1$ command (SHFT) then $\bigcirc$ .  Press EXE to reveal the result.	(Mat A) <sup>-1</sup> $\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{bmatrix}$ O Mat Mat Dat Try QUE D	Note that the brackets are not required here.  Also note you can use the power key ( ) and -1.  Mat A <sup>-1</sup> [ \frac{3}{2} - \frac{1}{2} \]  Mat MAL DEL TITH GUE   b
Determinant  det A  Press F3 (Det) then press F1 (Mat) then ALPHA and the appropriate letter.  Press EXE to reveal the result.	Det Mat A 2  Mat Mat Oet Trn Aus D	

### SYSTEMS OF LINEAR EQUATIONS

#### How do I solve systems of linear equations using EQUA?

#### Example:

Suppose there are three different types of corn, of which there are three bundles of the first, two of the second, and one of the third making 39 *measures*. Two of the first, three of the second and one of the third makes 34 measures. And one of the first, two of the second and three of the third makes 26 measures. How many measures of corn are contained in one bundle of each type?

(This problem originally was printed in *The Nine Chapters on the Mathematical Art*, an ancient Chinese mathematics book.)

We can represent this problem as a system of linear equations, where the number of measures of each of the three types of corn can be represented by x, y and z respectively:

$$3x + 2y + z = 39$$

$$2x + 3y + z = 34$$

$$x + 2y + 3z = 26$$

To solve the system we can do the following.

#### PROCESS

ACTION	DIAGRAM	COMMENTS
Press (MENU) and enter EQUA mode (SSO)	MAIN MENU WASHINGTAT PART IS SHIP SHIP SHIP SHIP SHIP SHIP SHIP S	
Press <b>F1</b> to select Simultaneous.	Equation  Select Type F1:Simultaneous F2:Polynomial F3:Solver BIMU 2017 SOLU	
Use the function keys, F1 - F5, to select the number of unknowns. For this example, there are three unknowns. Press F2.	Simultaneous No Data In Memory Number Of Unknowns? 2 3 4 5 6	
Key in values. Press EXE after each entry.  Once complete, press F1 to solve.	anX+bnY+CnZ=dn	
The solution will be displayed.	anX+bnY+CnZ=dn  **[\$929] Y 4.25 21 2.15]  REPT 26 26 26 27 26	Note that as each element is highlighted, if possible the fractional value of the number will be displayed in the bottom right of the screen.

⚠ Note that if the system of equations being solved has no solutions, or infinitely many solutions, the calculator will return Ma ERROR. It will not tell you which of the cases it is. Row operation will need to be used to determine this.

How do I solve systems of linear equations using row reduction? Solve the following system of equations:

$$3x + 2y + z = 39$$
  
 $2x + 3y + z = 34$   
 $x + 2y + 3z = 26$ 

# SOLUTION

ACTION	DIAGRAM	COMMENTS
Enter the RUN·MAT application (학교).  Press F3 to enter the MATrix mode.	MAIN MENU SITURE STATE OF STAT	
Define Mat A to be a three by four matrix, of the coefficients and constants of the system	A	
Press (EXIT).  Press (OPTN), select MAT (F2) and then press (F6) to see more options.	Com oim sill ref rei o	
Use the command:  Ref (F4) Mat A  to reduce Matrix A to row echelon form.	Ref Mat All	Note: when 'Mat' cannot be seen above one of the F keys, it can be found by pressing [HIF] then [2].
	$\begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{3} & 13 \\ 0 & 1 & \frac{1}{5} & \frac{24}{5} \\ 0 & 0 & 1 & \frac{11}{4} \end{bmatrix}$ Iden Dim Fill Ref Ref D	

How do I solve systems of linear equations using matrix operations?

Example:

Solve this system of linear equations:

$$3x + 2y + z = 39$$
  
 $2x + 3y + z = 34$   
 $x + 2y + 3z = 26$ 

This information can also be represented by a matrix equation in the form Ax = B where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 39 \\ 34 \\ 26 \end{bmatrix}.$$

Hence we can find x since  $x = A^{-1}B$ .

#### **PROCESS**

ACTION	DIAGRAM	COMMENTS
Enter the RUN-MAT application (ﷺ).  Press F4 to enter the Math mode and then press F1 to bring up the MATrices options.	MAIN MENU  MAIN MENU	If you define the matrices as A and B first, then the same calculation can be made using Mat A and Mat B. This is shown below.
	□ □	
Press $F2$ to bring up a 3x3 template.  Use mxn ( $F3$ ) and $x^1$ to enter $A^{-1}B$ and press $EXE$	[3 2 1] -1 [39] 34	Mat A <sup>-1</sup> ×Mat Bl
	2×2 3×3 m×n	Mat Mat Det Trn GuS D

Note that if the system of equations being solved has no solutions, or infinitely many solutions, the calculator will return Ma ERROR when trying to compute the matrix product as the inverse of A will not exist. It will not tell you which of the cases it is. Row operation will need to be used to determine this.

# POLYNOMIAL EQUATIONS.

# SOLVING POLYNOMIAL EQUATIONS

QUESTION

How do I find the solutions to a polynomial equation?

Example:

Solve of this equation:

$$-x^3 - 8x^2 - 5x + 14 = 0$$

# SOLUTION

DIAGRAM	COMMENTS
MAIN MENU WWW.  RUHHMISTAT eact ssm  \$410 BES BETTS  GRAPH DYNE TABLE RECUR  CONICS CONFROM TYN  BES BES BETTS  CONICS CONFROM TYN  BES	
Equation	
Select Type F1:Simultaneous F2:Polynomial F3:Solver BMM 2007 BMLV	
Polynomial No Data In Memory	
2 3 4 5 6	Note the form that the
SOLV DEL CLR EDIT	equation must be expressed in to determine the values of a, b, c and d for correct entry is displayed at the top of the screen.
aX3+bX2+cX+d=0 c -1 -8 -5	
SOLU DES CLR EDIT	
aX3+bX2+cX+d=0	Note that all solutions of the polynomial equation are given.
a -2 3 -1 1 REPT	We suggest that when solving polynomial equations it is wise to graph the polynomial first. You will then be able to see the x-intercepts (roots/solutions to the equation.)
	MAIN MENU   MAIN

The End. Good luck with your exam and your future.