Modelling the behaviour of springs

Type 3 – Mathematical Modelling.

(Work in this assignment will be assessed against Criteria A,B,C,D and F. Hence you are expected to use either MS Excel or a graphics calculator in you work.)

In this assignment you will develop a mathematical model for the behaviour of a spring system that you have designed.

Task One – System Design

Your teacher will supply you with a variety of springs and masses. You are to design *two* systems. For instances you may choose to use the same spring in each system and vary the mass attached to it, or you may use two different springs and attach a different mass or something different again.

Describe your two systems clearly using appropriate diagrams to support your description.

Clear description of the differences and similarity of their systems, ie.

- *weight same or different*
- *amplitude same or different*
- *spring same or different*

Task Two – Data collection

Use the Casio EA-100 Data Logger and the *Springy Things* program on your calculator to collect data that illustrates the motion of your spring systems.

Display the data both in tabular form and graphical form, ensuring you clearly describe the variables involved.

Two tables of data and two graph required.

Accept hand drawn or gcalc dumps or Excel graphics.

Inappropriate labelling (or absence of labelling) should be penalised

Task Three – Prediction

Before you perform any analysis, and assuming that the **first system** that you designed could be modelled by:

$$d = A\cos B(t + C)$$

where d is the displacement of the bob about the mean position, t is time in seconds and A,B and C are parameter, **predict the model appropriate for the second system** in terms of parameters A,B and C and others of your choice.

Answer depends on their systems.

Reward any answer that conveys the correct general relationship between the systems (see Task One)

- *amplitude*
- *period and phase*

for example, if the lift height was halved in System 2 then students we have A/2 and similarly for A, B and C.

Task Four – Model formation-by mind

Use your knowledge of trigonometry and your graphics calculator ability to take a snap shot of a graph of collected data to form an appropriate **cosine** model for each system. Explain clearly how you used your knowledge and the electronic technology to produce the models.

Students are required to have done a trial and error approach here. They may have started with y = cos x and drawn it over the top of the data (see Task 2), seen that it did not come close to the data and then fiddled with A, B and C. They must make clear the process they went through, not just have supplied an answer.

Task Five – Model formation-by technology

a) Use the **sine** model fitting capability of your calculator to find an appropriate **sine** model for each system.
b) Show that cosine model you formed, and the sine model produced by the calculator are approximately equivalent.
c) Explain why your calculator not have a cosine fitting feature?

a) This is the result they get when using the 'black box' sine fitter on the calculator

b) Starting with the cos model from Task 4, they should use the identity cos (theta) = sin (90-theta) and show all steps n the transformation to reach a result ***approx.*** *similar to that given by the machine.*

c) Hardly necessary when you can use the simple identity above.

Task Six – Extrapolation

Use your model to predict the displacement of the bob in your ***first*** system <u>one hour</u> after you put it into motion.

Comment on the integrity of such a prediction, what does it suggest about the model that you have formed?

Students should let t=3600s (pv their model has t in seconds).

Students are required to note that this prediction is silly as it suggests that the bob has continued to 'bob' for an hour. They should add that air resistance and other frictional forces act upon it and it would have been (seemingly) motionless after 1 hour.

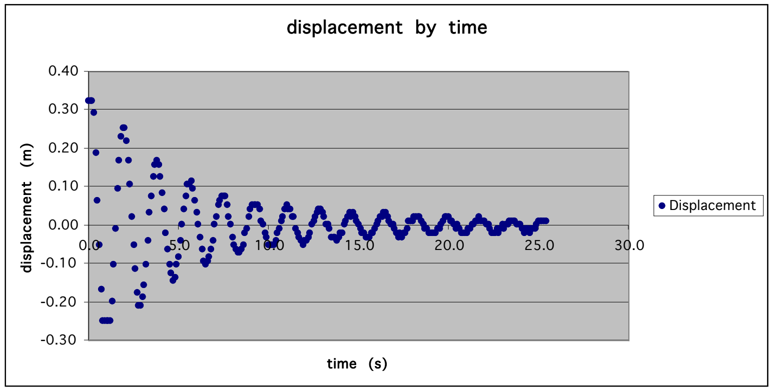
Task Seven – Working with your model of the <u>first</u> of your systems

a) Working within the limitation of your model, use it to determine the when the bob passes through its mean position.
b) Find the derivative of your model
c) Find the time when the derivative is zero and interpret the result in terms of your system
d) Draw a graph of your model and the derivative on the same set of axes
e) Determine the time when the derivate is of maximum value and the corresponding displacement of the bob. Interpret the result in terms of your system.

a) Students must set displacement to 0m and solve the resulting equation – any methods acceptable – penalise non-general solutions. Also penalise students who just write an answer down with no equation. – Solution by ET OK.
b) Student must apply the rules of differentiation correctly.
c)As of a) – penalise non-general solutions (especially as 2 families of solutions are required here).
Interpretation is that the bob is at the max/min positions when the derivative is 0.
d) Obviously the graphs need all appropriate labelling, and the max/min of f(x) must match with the intercepts of the derivative, also the inflections of f(x) must match the max/mins of the derivative – penalise inaccurate graphs if draw by hand.
e) These ***times*** *can be achieved by using ET.*
Interpretation is that this is where the bob goes through the mean position (and/or accept velocity is at a maximum

Task Eight – Refining the model

The data attached (see over) was collected from a spring system designed by Mr H. The bob had a piece of cardboard to exaggerate the dampening effect of air on this system. This data can be transferred into your calculator.



a) Develop a model that effectively predicts the behaviour of this system.
b) Determine the time when the bob is travelling at its maximum velocity.

a. Here the students are expected to use their knowledge of 'trial and error' fitting they used earlier. They also have to realise that more than a trig model is needed here and that some 'reducing' function is required.

Some may try the product of a rational function and a trig – this is a problem.
Some may try the product of an exponential and a trig.

Student should explain their choice.

One possible "close enough' solution is:

- *y=(0.8^x) 0.35 cos3.2x*

Students who give something like y=(1.6/x) 0.35 cos3.2x should be penalised.

Accept something close to the dot point solution – or something else if it is ***plausible****.*

b. Students will have hopefully made the connection back to the last task here. Students should describe clearly the fact that max vel means going through the mean position ***or*** *that it is when acc (second derivative) is zero and sign change (or a graph using technology.*
Students could achieve an answer by using technology to graph the derivative and find the max or use traditional methods – either way so long as the solution logically.
Penalise students who simply give answers.