Generosity

Seeing the world through multiplicative eyes in 10 'easy' steps

Anthony Harradine

Potts-Baker Institute, Prince Alfred College

aharra@pac.edu.au

First formal 'write-up' mid 2014.

This sequence was developed over a 4-year period, starting out with a Year 9 student who did not have a grasp of the 'for every', or rate, notion then moving onto uses in other ways.

Warning

This document captures a developmental sequence that can take place over an extended period of time. For a student who is not yet able to think multiplicatively, this development may not be able to take place in the space of a few days.

Note

Questions 1 to 4 are meant to be presented without any visual aid. Post question 4, it is intened that dot arrays presented in a particular configuration be used to support thinking. These can be accessed by contacting the authour by email at aharra@pac.edu.au

Question 1
Linda offers to give her cousin Charlie 20 jelly snakes.
Alishka offers to give her cousin Charlie 30 jelly snakes.

Who is more generous, Linda or Alishka?

What might happen?
Set up students in groups of three each student has a book and pen and one is elected the speaker, while each are scribes.
Explain they must provide an answer with a reason. Allow them time to think about the question and then ask each speaker to provide the group answer (the reason they are required to write is so they do not change their stated answer when listening to other's answers. Although when answering verbally, they may say, "We thought that …….. but listening to ……. "

It is likely many groups will provide the superficial answer, Alishka. The will not think about the possibility that Linda has only 20 snakes and Charlie might have 200. Many kids are able to observe the immediately visible only and not see what is related but cloaked (as Ellen in Year 7 told us).
If they do note the cloaked, then GREAT, if not then I have to ask a question to draw their attention to this critical aspect. For example, "Can you think of situation that would mean Linda is the most generous?"
Conversations will then follow.
We live in an absolute world, comparisons are no just there to be made, we need to actively make them. Our minds most naturally work in an absolute (additive) manner – not comparative/relative, where numbers are concerned.

Key idea from question 1
The amount given away is not the only thing to consider, the amount a person has may also be important to consider.

Question 2
Linda has 50 jelly snakes and offers to give Charlie 20 of them.
Alishka has 50 jelly snakes and offers to give Charlie 30 of them.

Who is more generous, Linda or Alishka?

What might happen?
Most students will be happy to say Alishka is the most generous because they have the same amount and she gave away more. **Some will focus on how many each person has remaining**.
However, expect to hear someone say, "maybe Linda has 8 brothers and sisters and Alishka is an only child and so ……. While might seem annoying, it is a critical aspect to address, as context does matter. So when such thigns are raised, just add a condition to the statement of the question that equalises all aspects except for the amount given away and the total they have.

Key idea from question 2
Larger give away amount (or the smaller remaining amount) when the 'base' amount is the same implies greater generosity (all else being equal).

Question 3
Linda has 60 jelly snakes and offers to give Charlie 40 of them.
Alishka has 80 jelly snakes and offers to give Charlie 40 of them.

Who is more generous, Linda or Alishka?

What might happen?
Hopefully this one will now be very uncontroversial in terms of determining who is the most generous, but there is an interesting aspect to be considered – outlined in the key idea stated below – the inverse idea present should be highlighted.

Depending on the learners – there is a opportunity to link this to fractions – but at this point, for many students it is too early to do this and it will spoil the development – if they make the connection, great. I would personally wait until they suggest that fractions might be helpful. If students have poor fraction skills, they will not mention it and I would wait to make this connection until a little further on.

Key idea from question 3
If the give away amount is the same then the smaller of the two base amounts implies greater generosity (all else being equal).

Note that at this point, all 'comparative thinking' has not required the learners to do 'hard' calculations – the comparisons are almost non-calculative.
Also, most students (and many adults) are thinking about generosity from the point of view of the absolute amount given away or that remains, and this way of thinking is limited in scope – as they will come to see.
The aim from here on is to force the students to start thinking about the notion of 'for every'. i.e. for every 10 Fred has he gives away 2 – but how do we do that by questioning as opposed to telling?

Question 4

Linda has 40 jelly snakes and offers to give Charlie 30 of them.
Alishka has 90 jelly snakes and offers to give Charlie 80 of them.

Who is more generous, Linda or Alishka?

[Note that changing the number 80 to 75 makes this question the 'same' but a bit more challenging. We are aiming with this question to raise a conflict in the decision making of the student so they will self-critique their decision.]

What might happen?

It is critical to realise that there is, perhaps, no correct answer here – just two different ways-of-thinking about this situation. This is the 'gold' here as we are hoping to raise a conflict in the thinking of the student as initially they may say they are equally generous (as they both have 10 left), but then some will quickly question themselves on this decision. Read on.

Some students will think they are equally generous – as they are each left with 10 snakes. For these children my job is to ask them if they can see a way of arguing that one is more generous than the other.

Some will, hopefully, say that Alishka is more generous as she is giving away more of her 'base' – they might twig that 30 of 40 is ¾ of a pie and can easily 'see' that 80 of 90 is 'more' but may not be able to clearly explain why and may even second guess themselves as if they visualise 30 from 40 as three quarters of a pie, they may not be able to reconcile how to think about 80 of 90 – they may want to make a bigger pie and then ……

It is at this that the point the fun begins and we are going to develop (or reinforce) the idea that 80 or 90 is 'more' than 30 of 40 even though there are 10 left in each case.

Some may calculate that 30 from forty is 75% and 80 from 90 is greater than 75% ….. it is interesting to try and see if these students 'just' know to do that sort of calculation with this sort of problem or they have a sound way to think about comparing things with different bases – i.e. they are sound multiplicative (for every) thinkers. Fewer humans in the populous are, than are not.

It is likely that it is a good time for the bell to go now and leave this for the kids to ponder till next time you meet.

Key idea from question 4

There are two ways to think about this and one easy to think about and one is more challenging (if you have not already mastered it).

Question 5
Linda has 30 jelly snakes.
Alishka has 60 jelly snakes.

How may jelly snakes should each person offer Charlie for them to be considered equally generous?

What might happen?
Hopefully the students will be able to supply two answers to this question.
L,A = 30, 60 or 29, 59 or ....
(one way of thinking about generosity)

Or (and of course the fun part .....)

Many will be able to 'see' that L = 15 and A = 30 will make sense because one has twice the other and so if one gives away twice what the other does then they will be equally generous.

Of course some students may well offer alternatives – like L = 10 and A = 20 – ask them why this is correct?

If discussion does not lead to it, my job is to ask if
L = 9 and A = 18 would mean they are equally generous? And if yes, why? If not, why not? Discussion here needs to lead to the following way-of-thinking:

Think about the base amount as being made of small clumps of a nice number – 10 in this case – and the same number for each (this is exploiting the idea developed in question 2.

So:
** 30 is three groups of 10
** 60 is 6 groups of 10
and so if each person gives way the same number from each of their groups of 10 we could say they are equally generous.

If most likely will be critical here to be using images to illustrate this – dot arrays or ....

So what will constitute equal generosity when thinking about it this way?
L = 3 and A = 6,
L = 6 and A = 12
etc.

It maybe here that kids start to realise that if you are allowed to cut snakes into bits then there are even more 'correct' answers.

Key idea from question 5
When the base amount is different, break them into groups of some "nice" number.

Question 6
Linda has 35 jelly snakes.
Alishka has 75 jelly snakes.

How may jelly snakes should each person offer Charlie for them to be considered equally generous, in the for-every sense?

What might happen?
Hopefully the students realise that 10 is not a nice number here and choose 5.

Key idea from question 6
The nice number is dependent on the number of snakes each person has.

Question 7
Linda has 49 jelly snakes.
Alishka has 63 jelly snakes.

How may jelly snakes should each person offer Charlie for them to be considered equally generous, in the for-every sense?

What might happen?
Hopefully the students will choose 7 as the nice number.

Key idea from question 7
The nice number is one that divides into each number.

Question 8 (revisiting Quest 4)
Linda has 40 jelly snakes and offers to give Charlie 30 of them.
Alishka has 90 jelly snakes and offers to give Charlie 80 of them.

Who is more generous (in the for-every sense), Linda or Alishka?

What might happen?
You might stop teaching forever if they do not get it right!

Key idea from question 8
We can determine when they are equally generous – so we can now judge who is more generous in the 'for every' sense.

Question 9
Linda has 7 jelly snakes and offers to give Charlie 4 of them.
Alishka has 11 jelly snakes and offers to give Charlie 6 of them.

Who is more generous (in the for-every sense), Linda or Alishka?

What might happen?
All hell might break loose now!

What we hope can be developed now is that we can see that we can multiply as well as divide to make an equally base.

Imagine if:
Linda had 77 (11 lots of 7) then she would give away 44 (11 lots of 4).
Alishka had 77 (7 lots of 11) then she would give away 42 (7 lots of 6).

So Linda is more generous.

Key idea from question 9
Prime numbers suck!
But we can multiply up to reach equal bases.

Question 10
Linda has 31 jelly snakes and offers to give Charlie 17 of them.
Alishka has 57 jelly snakes and offers to give Charlie 30 of them.

Who is more generous (in the for-every sense), Linda or Alishka?

What might happen?
They might revolt.

Time to think very mathematically and say we should be able to multiply up and down to some base size that we all agree up on – so we can automate this difficult process.

Lets just pick a nice number – I herby decree it is 100.

HM lots of 31 is 100?
What times 31 = 100?
Well 100/31 of course!
So Linda would give away 100/31 lots of 17 = 54.8

Alishka would give away 100/57 lots of 30 = 52.6

So ……

We might also try to develop this way of thinking:

17/31 = x/100
30/57 = y/100

and then consider ways of proceeding – the ways will depend on the students.

Key idea from question 10
Prime numbers suck more than I thought!
OMG it is percentages!

Note that this can be used to show how mindlessly being able to do a calculation can support the ability to understand this central and important understanding.

Also the ability to do multiples and mental computation ……….

This is a really nice development and example.