Mathematical Studies

Pre-Course - Ways of Thinking

Day 3 Summary and Day 4 problems

From Day 3 we have the following two big ideas

BI #1 – integration (aka the 'area' function or the quantity from rate function)

If we have a function $f(x)=3x^2$ and we want to calculate the area under the curve (to the x-axis) from $x = k$ to $x = x$ then we can do so using:

Let the total area under $f(x)=3x^2$ from $x = k$ to $x = x$ be denoted $S_k^x 3x^2$ then:

$$S_k^x 3x^2 = x^3 - k^3$$

(refer to your pattern-full table from yesterday)

More generally we think that:

$$S_k^x n+1x^n = x^{n+1} - k^{n+1}$$

Even more generally we think that:

$$S_k^x ax^n = \frac{a}{n+1}x^{n+1} - ???$$

Now substituting the 'correct' terminology:

$$\int ax^n = \frac{a}{n+1}x^{n+1} + c$$

Note the k and the x have gone! And the ??? are a c, what is going on?

Well, I have *not* told you the full story! But, a pretty good part of it that will help you to understand the rest – when you meet it.

Some examples:

$\int 2x^3$

$= \frac{2}{4}x^{3+1} + c$

$= \frac{1}{2}x^4 + c$

Note: Dear reader, you may be horrified with use of incorrect symbolism – but please understand these were the forms we developed in an informal way to describe the ideas. 'Proper' forms were developed in the formal classes.

BI #2 – differentiation (aka slope function or the rate from quantity function)

If we have a function $f(x) = x^3$ and we want to slope of the tangent to this curve at some particular value of *x* then we can do so by using:

Let the slope of the tangent at some particular value of *x* on $f(x) = 3x^2$ be denoted $\frac{\mathrm{d}(x^3)}{\mathrm{d}x}$ then:

$$\frac{\mathrm{d}(x^3)}{\mathrm{d}x} = 3x^2$$

(refer to your pattern-full table from yesterday)
I thinking about this as the 'slope' function.

More generally we think that:

$$\frac{\mathrm{d}(ax^n)}{\mathrm{d}x} = anx^{n-1}$$

This is an easier story to tell than the integration story!

Some examples:

$$\frac{\mathrm{d}(\frac{1}{2}x^4)}{\mathrm{d}x}$$
$$= \frac{1}{2} \times 4x^{4-1}$$
$$= 2x^3$$

BI #3 – Fundamental Theorem of Calculus

Notice anything?

This discussion turned out to be very cool. Ben Siebels – said – WHAT – explain that again, so I drew a picture two separate functions – one the derivative of the other and showed what on one was computed on the other – it was a very cool moment.

Problem #1

Graph the following functions:

$$y = 2x^2 + 4x + \frac{1}{2}$$
$$y = -2x^2 - \frac{1}{2}$$
$$y = 4x^2 + 6x + 1$$

Notice anything cool? Describe it. Then determine one more function that fits in with this bunch. If you can do that, find all the functions that fit in with this bunch.

Problem #2

Consider all cubic functions of the form

$$y = ax^3 + bx^2 + cx + d$$

a) Find the derivative of this massive set of cubic functions

b) Some cubic functions have two turning points – draw a sketch to show you understand what this means.

c) Find the x co-ordinate of the turning points of all the cubic functions that have them

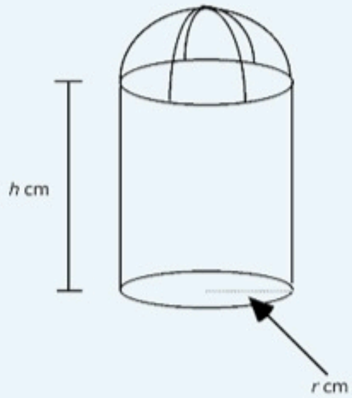
d) Illustrate some specific cases that have turning points and other that do not using your answers above.

Problem #3

Use the same method we did yesterday by drawing tangents on curves using electronic technology, try to discover the derivative (slope function) for

$$y = 2^x \text{ and } y = 3^x$$

Problem #3



The engines that are used in lawn mowers are called single cylinder engines. The piston moves up and down within a cylinder. Above the cylinder is a hemispherical combustion chamber. The diagram opposite shows the structure.

This type of engine is commonly called a 100cc engine. This refers to the volume of the cylinder and hemisphere combined. The cc stands for cubic centimetres (cm^2).

The problem for the designer of such an engine is to set the dimensions (*h* and *r*), given there are an infinite number of possibilities.

To get a feel for the infinite number of possibilities use the interaction called ***Cyl-hemisphere visualiser1***. Remember that the volume remains constant at 100cc.

Sam, an engineer, decides that the best dimension for *h* and *r* would be the ones that made the surface area of the structure as small as possible. Then the cost of producing the structure could be less.

Do the following to help Sam build a mathematical model that will assist in finding the *h* and *r* when the surface area is minimised.

1. Show that the surface area (S) of this structure is given by $S = 2\pi r(r+h)$. Comment on the significance of this finding.
2. Show that the volume of the structure is given by: $V = \pi r^2 \left(h + \frac{2}{3}r\right)$
3. If the capacity of the above structure is 32π cubic centimetres (which is close to 100cc), show that the height of the cylinder (h) is given by: $h = \frac{32}{r^2} - \frac{2}{3}r$
4. Hence show that the surface area of the structure can be expressed by the following function: $S = 2\pi\left(\frac{96+r^3}{3r}\right)$
5. Find the value of the radius that will provide the maximum surface area.
6. Find the height that corresponds to this radius.