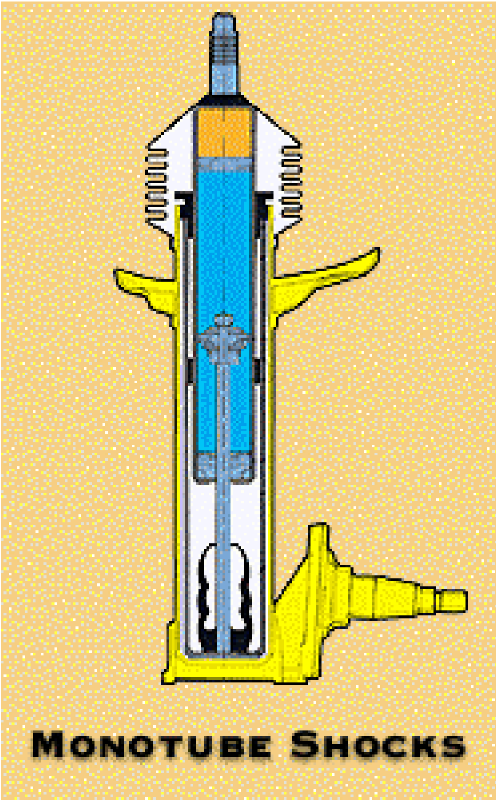
Simulating a Shock Absorber

Introduction



Shock Absorbers have many applications. The most obvious application is their use in motor vehicles. They are attached to the wheel of the car and the body of the car. Inside the Shock Absorber is either air or oil that acts as a dampening agent to the shock that bumps in roads and the like put on the vehicles wheels. Essentially, dampening the shock means that instead of the wheel bouncing up and down repeatedly and perhaps uncontrollably, the wheel only bounces up and down a few times with decreasing severity. Such motion is called '***dampened simple harmonic motion***'.

A great deal of mathematics is required to be able to design a successful Shock Absorber. We can not experiment with actual Shock Absorbers, but we can simulate a shock absorber and then experiment with the system so we can predict outcomes from the system should we change the value of certain variables. Our Shock Absorber is simply a spring with a mass applied that will act as the shock. The dampening will be supplied by a piece of cardboard positioned at the base of the mass cradle. Hence this is an air-dampened system.

Your Tasks

Task 1 – A qualitative description

Watch your teacher demonstrate the simulated shock absorber. Consider the differences and similarities to this system and a cars shock absorber. Think about how the *position of the mass varies* over time when the mass is lifted to the maximum height possible and let fall freely to supply a shock to the spring.

Also watch as a number of different air dampeners that have different surface areas ($100cm^2$, $200cm^2$, $300cm^2$, $400cm^2$ and $625\ cm^2$), are used on the simulated shock absorber.

At the start of the write up of this project:

- **describe how you think the amplitude of the mass's motion changes over time for just one dampener. [Amplitude is the maximum distance the mass reaches from its rest position]**

- **describe the effect that you feel dampeners of different surface area will have on how the amplitude changes over time.**

Task 2 – A quantitative description

Form a team of three or four students. Each team has two tasks in this section:

- research dampened simple harmonic motion and discover what ***form*** the change of the amplitude of the mass's motion should exhibit.

- working with only *one* dampener (a different one to all other groups) develop an algebraic rule that appropriately models the change in *amplitude* of the mass's motion as a function of *time*.

Once you have completed this task, share your findings with the other groups in the class.

Task 3 – Investigating a claim

- As an individual, use the findings of your group, and the other groups, to make a conjecture about the relationship between the decay in amplitude and the surface area of the air dampener.

  Remember that a conjecture is a statement based on some evidence, not just a guess.

Tools at your disposal

- The library
- The Internet
- Data logging equipment that can measure displacement (and a demonstration of how it works)
- Your graphics calculator
- A spreadsheet
- Anything else you can think of.

Information for the teacher

Experimental set-up

The choice of spring is critical. You need one with the ability to extend a considerable way when around 100 grams of mass is added. We have used a 25 cm spring that extends to approximately 50 cm when 100 grams is applied. We have also had success with a series of rubber bands looped together. Five size ten bands works quite well.

You should experiment with the spring,mass and dampener combination you have to get a period of about no less than 1 second but no greater than 2 seconds.

The distance between the motion sensor and the dampener is also critical. If too small the data-logger will not perform accurately, the same will happen if it is too far away.

We have found that if the distance between the motion sensor and the dampener is in the order of 70 cm good results are returned.

The air dampeners can be made of cardboard or foam and can be of whatever area you desire.

Collection of data

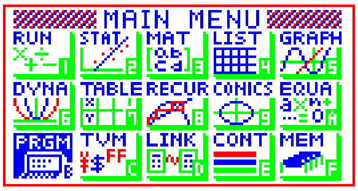
Students can use the program called CASIOLAB (installed on the Casio 9850GB PLUS) to drive a Vernier Motion Sensor connected to the Casio EA-100 data logger. This will allow them to collect measurements of the distance from the motion sensor to the dampener. Later they can use a program called LCHANGE.

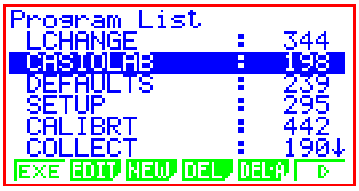
Both programs are self-explanatory once they are running.

To learn how to load the programs into your calculator from another calculator, or from a computer, go to the LUMAT website (http://www.lumat.net) and download the document called Using the FA-123 Program Link. If you do not have the Program Link it can be downloaded from the website as well.

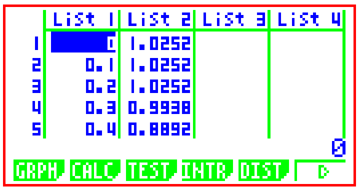
Once loaded connect the data logger into the sonic channel of the EA-100 and the data-logger to the 9850GB PLUS with a SB-62 cable.

CASIOLAB is launched by selecting the PRGM mode, highlighting CASIOLAB and then pressing EXE.





Follow the instructions there after. When asked to select a type of data collection, choose **non-real time** and then choose to collect data samples at intervals of **0.1 seconds**. Collect the maximum of **255 data points**

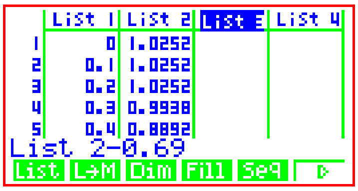


Once the data (time and distance between sensor and dampener) collected has been transferred into List 1 and List 2 of the calculator analysis may begin.

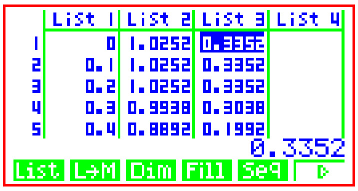
Data Analysis

The data seen in List 1 and 2 above were collected using the 625 square centimetre dampener. This is downloadable from the LUMAT website.

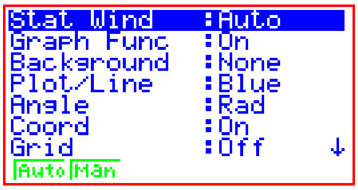
When at rest, the distance from the dampener to the data-logger when the spring was in its rest position was 69 cm. Hence, to determine the mass's distance from its mean position using the data we have collected, we need to subtract 0.69 m form each value in List 2.
Place the cursor in the top of List 3 and then press OPTN then LIST (F1) and then List 2-0.69.



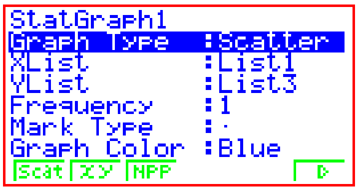
Press EXE and List 3 data will be calculated. These are the actual displacement values of the mass from its mean position.



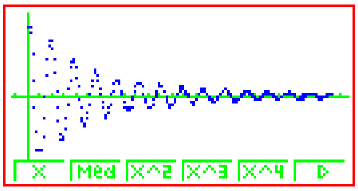
Press SHIFT and then SETUP to ensure that the settings are as shown below.



It would now be good to produce a graph of amplitude vs. time. Press EXIT until you return to you see the GRPH (F1) option and select it. Then use SET (F6) to set up GPH 1 as shown opposite.



Press EXIT and then GPH 1 (F1) to produce the graph. It looks like a *beautiful* case of dampened SHM.

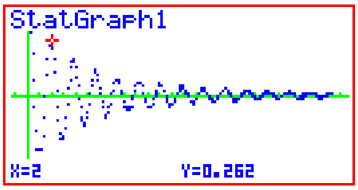


We are now really only interested in the peak values, that will give us the ability to model amplitude over time.

To extract the data points that correspond to the peaks we can use a small and rather simple program called LCHANGE (list change). The code is as follows:

```
"How many pairs are"↵
"to be transferred"?→M↵
Seq(0,X,1,M,1)→List 4↵
Seq(0,X,1,M,1)→List 5↵
1→N↵
Lbl X↵
If N=1↵
Then "First row # to be"↵
"transferred is"?→A↵
List 1[A]→List 4[N]↵
List 3[A]→List 5[N]↵
N+1→N↵
Goto X↵
IfEnd↵
"Next row # to be"↵
"transferred is"?→A↵
List 1[A]→List 4[N]↵
List 3[A]→List 5[N]↵
N+1→N↵
If N=M+1↵
Then Goto K↵
IfEnd↵
Goto X↵
Lbl K↵
ClrText↵
"All done."↵
"Press MENU and select"↵
"STAT mode to proceed."↵
Stop↵
```

Before proceeding we need to identify the **row #** of each of the data pairs we want to extract. To do this you can either TRACE the graph or simply scan the table of values.

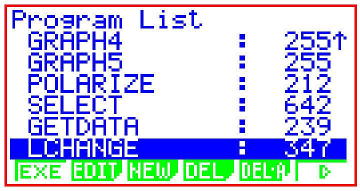


The table is probably a simpler process.

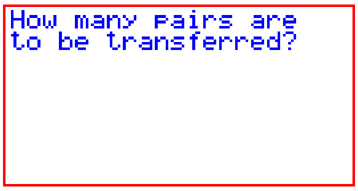
In this data set the peaks have the following row numbers (we have chosen only six):

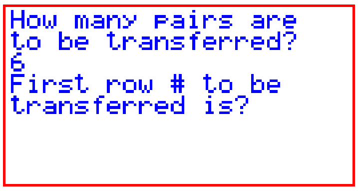
3, 21, 39, 58, 76,94

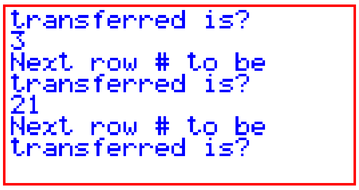
To start this program, enter the PRGM mode, highlight the program LCHANGE and press EXE.



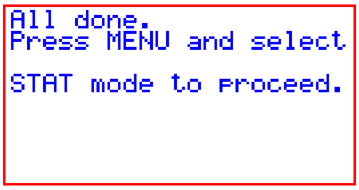
You should get the following self-explanatory screens.



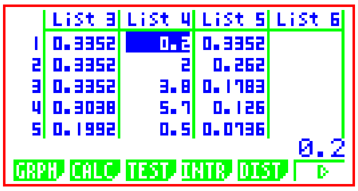




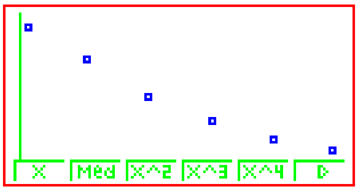
Proceeding and putting in all the six row numbers will result in the following:



Returning to STAT mode reveals success!

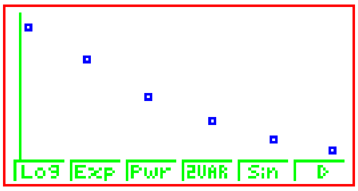


A graph of amplitude vs time (List 5 vs. List 4) reveals the following:

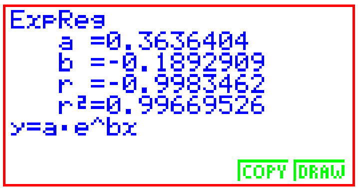


Fitting a model to this is now the task. Students should, via their research, find that an exponential model is what should be appropriate. **We would expect them to carry out log (base 10) transformations at this level, but we can do an 'e' fit.**

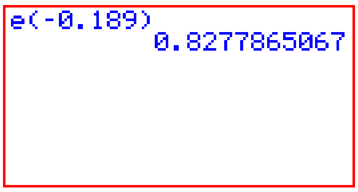
Press the continuation key (F6) and use EXP (F2) to fit an exponential model.



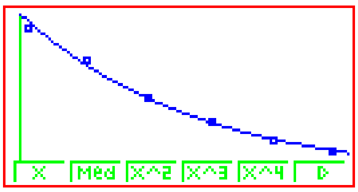
The following print out results.



A little further analysis returns the fact that the dampener is reducing the amplitude by about 17% per second.



We could also have drawn the curve of best fit using the DRAW (F6) option the previous screen. Looks good – balanced residuals that are not too large! (Could be sure by doing a residual plot).



We can now proceed with the rest of the dampeners and see what we find!

EARTHQUAKES

EXPONENTIAL GROWTH DIRECTED INVESTIGATION

The purpose of this investigation is to investigate the relationships between earthquake magnitude and energy released during earthquakes.

Read through the following information carefully before attempting any questions.

PART 1 THE RICHTER SCALE

In 1935 Charles Richter developed a scale that allows the measurement of the size of an earthquake using data from seismograph stations. Seismographs register the shocks and motions of earthquakes by measuring the amplitude of the resulting seismic waves in the earth. The overall tremor is recorded as a trace amplitude, measured in mm.

A standard earthquake is defined as one that will cause a 1 mm trace on a standard seismograph, 100 km from the epicentre of the earthquake. The magnitude of this standard earthquake is given a value of 3. To calculate the magnitude of any earthquake (based on the standard earthquake definition), it is necessary to know the distance from the epicentre and the recording station (containing the seismograph), and the maximum trace amplitude in mm. The nomograph shown in Appendix 1 can then be used to find the magnitude, M. This scale is called the Richter scale.

Using the nomograph:

The symbol Δ, refers to the distance between the epicentre and the seismic station and is measured in km. B, in mm, is the trace amplitude measured on the seismograph, and M is the magnitude of the earthquake. Using a ruler, line up the values, $\Delta = 100$ km, B = 1 mm and read off the value for M. You should get M = 3, since this is the standard earthquake magnitude for these values of Δ and B. To measure any earthquake, corresponding values of Δ, B and M lie on a straight line.

Task 1:

Investigating the relationship between magnitude, M, and the trace amplitude (mm) at a set distance of 100 km from the epicenter.

1. Use the nomogram in Appendix 1 to obtain a table of values for magnitude, M and amplitude, B, while $\Delta = 100$ km.

Magnitude, M			3			
Trace amplitude, B			1			

2. Use your graphics calculator and skills learnt from this section of work to find a relationship between magnitude and trace amplitude. Show all your reasoning and any relevant sketches and tables from your screen. Scales on the sketches please.

3. Check your rule found in part 2 with two more readings from the nomograph.

4. Read the following statement and comment, clearly and in detail, using your results from this task:

> " … each whole number increase in magnitude represents a tenfold increase in measured amplitude."

PART 2 ENERGY RELEASE DURING EARTHQUAKES

An earthquake occurs when there is a sudden release of energy. The total amount of released energy does mechanical work in moving masses of rock, generates heat as a result of friction, and radiates seismic waves. The amount of energy released as seismic waves can be approximated with the following formula:

$\log E = 1.4 + 1.5M$,
where E = the amount of energy released in waves,
and M = magnitude of the earthquake.

This formula is derived from many observations and experiments. It is therefore called an empirical model.

The energy calculated using the above formula is only a percentage of the total energy released by an earthquake. The energy stored as strain in any volume of rock will be:

$$E_{total} = \frac{1}{2} G\varepsilon^{2} V$$

Energy, E, is measured in ergs, where the erg is the standard unit of energy. G is a constant, related to stress (the force per unit area), called the elastic modulus. ε is the strain in the rock body, and is related to the ratio of the change in length of a rock body under compression and the original length. V is the volume of rock, measured in cm^3.

Task 2:

Investigating energy release and magnitude of earthquakes.

1. In 1964, an earthquake in Alaska damaged the homes and destroyed the business and commercial district of Anchorage. A seismograph in the USA at Berkeley, California, recorded a trace amplitude of 50 mm. Berkeley was measured to be 3000 km from the epicentre. The strain released in the region, ε, was 1.1×10^{-4} and the elastic modulus, $G = 3 \times 10^{11}$ units.

   (a) Determine the magnitude, M, of the earthquake.
   (b) If the distance between Boston and the epicentre is 5500 km, find the seismograph trace (mm) recorded in Boston.
   (c) Find the amount of energy released as waves in the earthquake.
   (d) Assuming that only 50% of the total energy released was in the form of waves, what would the total energy be in the area just before the earthquake?
   (e) A 20-kiloton atom bomb releases approximately 8×10^{20} ergs of energy. How many atom bombs is the total energy found in part (d) equivalent to?
   (f) Find the volume of earth affected by the total energy release in cubic kilometres.
   (g) Assume that the depth of earth affected was 10 km, find the area that was affected by the total energy release. (Volume = depth x area)

2. Read the following statement:

   "… each whole number step in the magnitude scale corresponds to the release of about 31 times more energy than the amount associated with the preceding whole number value."

   Comment on the statement, supplying detailed mathematical reasoning for your thoughts.

Paper Mountains

EXPONENTIAL GROWTH DIRECTED INVESTIGATION
(A little one)

Consider a piece of A4 paper, which is 0.1 mm thick. Cut it in half and stack it on top of the other half. The stack will therefore be 0.2 mm thick.
Cut this stack in half and stack it on top to obtain a pile 0.4 mm thick.

The process continues and is referred to as “cut and stack”.

Question: (Guess) How many cut and stacks do you think you would need to do, such that the stack produced will reach the Moon (approximately 400,000 km away)?

1. Draw up a table with the number of cuts, n, in one column and the thickness, T, of the resulting stack in another. Complete the table for n = 0 to 8 cuts.

2. Look at the values for thickness. What type of sequence is generated? What convinces you of your answer?

3. Using your answer from part 2, find the rule connecting n and T.

4. Use your graphics calculator and find log T for the values on n = 0, …, 8.

5. Now plot n against log T and find the linear relationship in the form logT = a.n + b.

6. Supply a sketch, with labeled axes, of the relationship found in part 5.
    (a) Should your sketch display a set of points or a continuous line?
    (b) Clearly explain your choice.

7. Return to the original question and find the number of cut and stacks required to reach the Moon. Make sure you show your method of solution.

Some test/exam questions and issues to consider

Introduction

These questions are not meant to form a test as such. They are meant to highlight the following facts:

- we can choose to make questions graphic calculator inactive(I), user choice(UC), active(A),
- the balance of I, UC and A is not yet clear for 2002 Stage 2– SSABSA is still to give some direction on this
- the words we use in writing our questions are of the utmost importance
- we need to consider 'old' questions value in the light of what technology we now have.

In each question we have highlighted key words that results in the question being I, UC or A. It is imperative the students become comfortable with the fact they will have to read questions closely.

Active questions seem scarce at the moment. Is that so bad? Having ET available has really allowed us to consider different learning and doing approaches. Our feeling is that there must be a very good reason to be pressing lots of buttons in a ***test*** situation.

Even if a question is made inactive, students may still determine the solution using the calculator as a check or at the start as a guide to the solution

The key will be to get a balance and not to test things over and over again in the same paper

1. Consider the sequence -6, -12, -18, -24, ...

   i) **prove** that these four terms are a part of an arithmetic sequence

   ii) **find** an expression for the general term, T_n of the sequence

   iii) **find** the 50^{th} term of this sequence

   iv) **find** the sum of the negative multiples of -6 from -18 to -300 inclusive.

1*. Consider the sequence -6, -12, -18, -24, ...

i) **prove** that these four terms are a part of an arithmetic sequence

ii) **find, using sequence formulae**, an expression for the general term, T_n of the sequence

iii) **find** the 50^{th} term of this sequence

iv) **find, using sequence formulae,** the sum of the negative multiples of -6 from -18 to -300 inclusive.

2. Rodney has just started in the road tarring business. His first contract is to tar 2000km of road in the outback. He is required to estimate the time the job will take him.

He figures that since he is learning about the job he will be able to do 4km in the first day and then *an extra 2km* than the day before on every day after that. **Determine** how long the job will take him.

2*. Rodney has just started in the road tarring business. His first contract is to tar 2000km of road in the outback. He is required to estimate the time the job will take him.

He figures that since he is learning about the job he will be able to do 4km in the first day and then *an extra 2km* than the day before on every day after that.

a) write down a sequence that models this situation
b) **Determine, using sequence formulae,** how long the job will take him.

3. a) find the value of x in each of the following

i) $2x = 12$
ii) $3x = 33$
iii) $4x = 64$
iv) $5x = 105$

b) consider the sequence 12, 33, 64, 105, ...

i) **show** that these four numbers are not part of an arithmetic sequence

ii) **using your result from a)** show **algebraically** that the terms on the sequence obey $T_n = 5n^2+6n+1$

3*. a) find the value of x in each of the following

i) $2x = 12$
ii) $3x = 33$
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b) consider the sequence 12, 33, 64, 105, ...

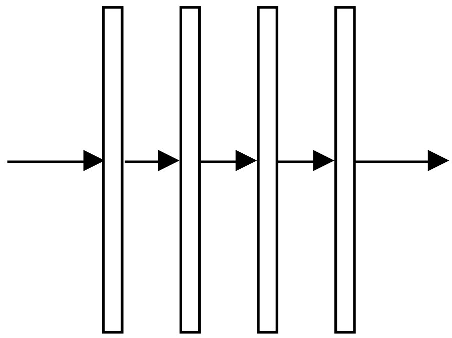
i) **show** that these four numbers are not part of an arithmetic sequence

ii) **show** that the terms on the sequence obey $T_n = 5n^2+6n+1$

4. When light passes through a certain type of glass, the light exiting the glass is of less intensity than when it entered. The intensity on exit is dependent on the thickness of the glass.

It was found that the intensity of light (I_n) leaving 'n' panes of glass 3 millimetres thick is given by:

$I_n = I_0 2^{-0.3n}$ where I_0 is the original intensity of the light source.



a) If the light passes through 4 panes of glass, **find** the intensity of the exiting light in terms of I_0.
b) **Determine** how many panes of glass are required to reduce the intensity of the light by 40% of its original value.
c) **Show that each of the four panes** reduces the intensity of the light by approximately 19%.
d) Now consider one very thick pane of glass. **Show** that the intensity I_x of the light 'x' millimetres after entering the pane is given by

$$I_x = I_0 (0.93)^x$$

e) **Explain** the meaning of the number 0.93 in the context of this problem.

4*. When light passes through a certain type of glass, the light exiting the glass is of less intensity than when it entered. The intensity on exit is dependent on the thickness of the glass.

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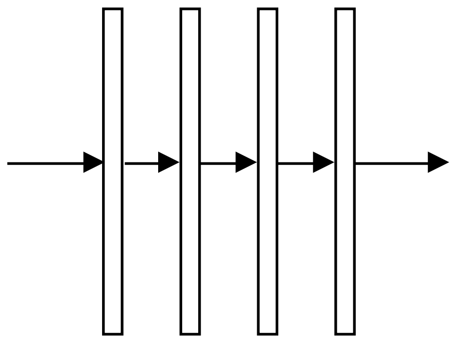

e) If the light passes through 4 panes of glass, **find** the intensity of the exiting light in terms of I_0.

f) **Determine algebraically** how many panes of glass are required to reduce the intensity of the light by 40% of its original value.

g) **Show algebraically (is the algebraically redundant) that each of *n* panes** reduces the intensity of the light by approximately 19%.

h) Now consider one very thick pane of glass. **Show** that the intensity I_x of the light 'x' millimetres after entering the pane is given by

$$I_x = I_0 (0.93)^x$$

e) **Explain** the meaning of the number 0.93 in the context of this problem.

5 Simplify

a) $27^{\frac{1}{3}}$ b) $16^{\frac{5}{4}}$

5* Simplify, using index laws and showing all steps of your working

a) $27^{\frac{1}{3}}$ b) $16^{\frac{5}{4}}$

6 a) Show that $5^{n-2} = \dfrac{5^n}{25}$

b) factorise $5^{n-2} - 5^{n+1}$

c) hence prove that $\dfrac{5^{n-2} - 5^{n+1}}{-\frac{62}{25}5^n} = 2$

7 **Solve** each of the following index equations

a) $16^{2-3x} = 8^{x+1}$ b) $3^{2x+1} = 2^{x+2}$

7* **Solve** each of the following index equations **using index/log laws showing each step of your working**

a) $16^{2-3x} = 8^{x+1}$ b) $3^{2x+1} = 2^{x+2}$

8 Evaluate the following

a) $\log_3 27$ b) $\log_{3p}(27p^3)$

9 Convert $\log_6 12 = A$ into an equation free of log terms.

10 Solve each of the following for *x*:

a) $\log_5(x+25) = 3$
b) $\log_{x+2} 9 = 2$

11 Prove that $-2\log(a^2b) + 3\log(ab^{-2}) + 1 = -\log(\frac{ab^8}{10})$

12 Write equivalent statements to the following that do not contain logarithms

a) $\log D = 0.669x - 1.845$
b) log K=5 log m+5